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THESIS

SOLVING OPERATIONAL MODELS OF INTERDEPENDENT INFRASTRUCTURE SYSTEMS

by

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December 2014

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SOLVING OPERATIONAL MODELS OF INTERDEPENDENT INFRASTRUCTURE SYSTEMS

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ABSTRACT

We formulate and solve a model of interdependent fuel and electric power infrastructure systems with explicit representation of the fuel required to run some electric power generators and the power required to heat and pump fuel. Our model determines a set of fuel and power flows that result in the minimum cost of operating both systems, including penalty costs for failing to deliver each material to each of several external customers. We then formulate models of each system separate from the other, and, for each system, represent each interdependent relationship as a demand node with associated penalties. We implement an iterative algorithm for solving various instances of the problem; the algorithm alternates between solving each system separately, and passing material requirements to the other model. We then evaluate how well our algorithm performs in comparison to the monolithic formulation. We conclude with suggestions for improvements to the algorithm.

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LIST OF ACRONYMS AND ABBREVIATIONS

FDM	fuel delivery model
MW	megawatt(s)
MWh	megawatt-hour(s)
PFM	power flow model
PCCIP	President's Commission on Critical Infrastructure Protection
RTS-96	IEEE Reliability Test System 1996

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EXECUTIVE SUMMARY

The United States is a highly developed country where every aspect of life depends on complex and interconnected infrastructures. As a result of the terrorist attacks of 11 September 2001, we are more aware of how a small number of attacks could cause significant damage to this country through its critical infrastructure systems. Stopping all such attacks would be ideal, but this is an unrealistic goal, particularly when facing “attacks” from Mother Nature. Instead, research efforts in this area have focused on how to use limited resources to protect infrastructure to minimize the consequences of a successful attack against a small number of vital components.

At the Naval Postgraduate School, current research is focused on the use of attacker-defender models that assume each opponent has limited resources, each has all of the pertinent information about the system, and each attack or defend optimally. The results from attacker-defender models give the infrastructure manager a clear understanding of the vulnerabilities in a system, and guidance for improving resilience, getting the biggest “reduction in bang” for their buck.

The most important part of those modeling efforts is the formulation of a mathematical model that captures the operation of the infrastructure or infrastructures involved and is of the appropriate fidelity to accurately assess the consequences of an attack on, and the effectiveness of defenses to, that system or systems. When there are multiple, interconnected systems, the models that consider interdependencies between infrastructure systems become more complex and difficult to solve.

In this thesis, we formulate a monolithic operational model of two interdependent infrastructure systems and solve it for a realistic instance of interdependent fuel and electric power systems. We then examine algorithms for solving these problems that iterate between smaller models of the individual systems, passing information about the requirements of each system to the model of the other system through shared data. This type of algorithm simulates the dynamic interaction between the two networks, and is a more realistic representation of how we might have to coordinate two large systems that are owned and operated by separate entities. Our focus is on examining the feasibility of solving multiple, less complex infrastructure models instead of one monolithic model that would be very hard to build and validate.

In many cases, the algorithm converges to a reasonable solution for the operation of both systems, but consumes resources unnecessarily, providing excess capacity (and therefore, waste) in the two systems. We provide a simple mechanism for “turning off” this excess capacity without affecting the system performance, and the solutions obtained are much more reasonable, and closer to the solution of the monolithic model.

In other cases, the algorithm provides solutions that are clearly (and significantly) suboptimal. For example, if the penalties for unmet demand on the interdependent systems are not large enough, then each system’s requirements might go unmet by the other system, leading to a rapid reduction in capacity for both systems.

These iterative algorithms are not guaranteed to converge to the optimal solution except in the simplest of circumstances; nevertheless, they are representative of the kinds of algorithms we would expect to use in situations in which competing infrastructure operators are reluctant to share full operational information with each other. With a reasonable approach to modeling, the demands between the systems, and by setting the penalties appropriately on failing to meet those demands, the algorithms can provide realistic operational plans and insight into the costs of operating interdependent systems without the need for a monolithic model that captures all aspects of the interdependent systems simultaneously.

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I. INTRODUCTION

The United States is a highly developed country where every aspect of life depends on complex and interconnected infrastructure systems. Business and industry, government agencies, schools and hospitals, and almost every other significant entity and activity rely on multiple types of infrastructures, from energy transmission systems to telecommunications, water, and transportation networks. As a result of the terrorist attacks of 11 September 2001, we are more aware of how a small number of attacks could cause significant damage to this country through its critical infrastructure systems. Stopping all such attacks would be ideal, but this is an unrealistic goal particularly when “attacks” can also come from Mother Nature. Instead, research efforts in this area have focused on how to use limited resources to protect infrastructure to minimize the consequences of a successful attack against a small number of vital components.

As the study of critical infrastructure systems has gained momentum, the focus has been centered on finding system weaknesses where an attack or series of attacks could cause the greatest damage, cost, or disruption, and on determining how best to improve resilience. At the Naval Postgraduate School (NPS), this work is focused on the use of attacker-defender models (e.g., Brown et al. 2006) of sequential games played between two opponents, a *defender* of who operates an infrastructure system and wishes to protect it, and an *attacker* who wishes to damage that system. These models assume that each opponent has limited resources, each has all of the pertinent information about the system, and they attack or defend optimally. The results from attacker-defender models give the infrastructure manager a clear understanding of the vulnerabilities in a system, and guidance for improving resilience, getting the biggest “reduction in bang” for their buck.

This type of research has been successfully applied to many different types of critical infrastructure systems. The most important part of those modeling efforts is the formulation of a mathematical model that captures the operation of the infrastructure or infrastructures involved and is of the appropriate fidelity to accurately assess the consequences of an attack on, and the effectiveness of defenses to, that system or systems.

There are many important infrastructure systems within the United States. Most of the current research usually assumes that any attack on a single infrastructure system affects that system only. Studies of single infrastructures can be quite useful; the simplification of the formulation makes the attacker-defender models relatively easy to solve and the results can provide insights into the vulnerabilities of that system. However, many situations exist where an attack on one infrastructure systems could impact other systems. For example, a natural gas power plant requires water, telecommunications, gas delivery systems, as well as a transmission system to completely function, and is therefore *dependent* on these other infrastructures that provide those functions. If any of those supporting infrastructures are attacked, the power plant might not be able function. Any model that explicitly considers these *dependencies* (or, in the case of infrastructures that support each other, *interdependencies*) would be larger and more complicated than a single, independent infrastructure model.

Much of the prior research in infrastructure defense has used mathematical programming models to determine the optimal operation of a single system both before and after an attack, assuming all other interdependent systems remain unaffected by an attack on that system. Recent efforts have modeled interdependencies between two systems in a single, monolithic model of operations that explicitly represents the connections between the two systems, but only small examples have been formulated and solved.

In this thesis, we formulate a monolithic operational model of two interdependent infrastructure systems and solve it for a larger, more realistic instance of the problem. We then examine algorithms for solving these larger problems that iterate between smaller models of the individual systems, passing information about the requirements of each system to the model of the other system through shared data. This type of algorithm simulates the dynamic interaction between the two networks, and is a more realistic representation of how we might have to coordinate two large systems that are owned and operated by separate entities; major utilities usually consider their models proprietary and would not readily allow a monolithic model to be created, especially with a neighboring and potentially competing utility. Our focus is on examining the feasibility of solving multiple, less-complex infrastructure models instead of one monolithic model that would be very hard to build and validate.

We summarize our results and offer some initial insights into the performance of these iterative algorithms in comparison to the optimal solution(s) obtained by the monolithic model, noting in particular when they are likely to work, and when they are likely to lead to extremely poor solutions. In many cases the algorithm converges to a reasonable solution for the operation of both systems, but consumes resources unnecessarily, providing excess capacity (and therefore waste) in the two systems. We provide a simple mechanism for “turning off” this excess capacity without affecting the system performance, and the solutions obtained are much more reasonable, and closer to the solution of the monolithic model.

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II. BACKGROUND

A. PREVIOUS WORK ON INTERDEPENDENT INFRASTRUCTURES

This chapter presents a brief history of critical infrastructure protection, followed by a short description of how some researchers are addressing dependency between infrastructures. We then describe baseline models for the operation of electric power and fuel transmission networks which serve as the basis for our model of interdependent operation.

1. Critical Infrastructure Beginnings

Although 9/11 was a highly publicized example of the importance of critical infrastructure protection, the concept of homeland security dates back to the creation of the National Communications System (NCS) in 1963, after communication problems between the U.S. and USSR further threatened the Cuban Missile Crisis (Lewis 2006). It was then that U.S. leaders knew how failures of critical infrastructures could have widespread and lasting effects. At that time the major concerns centered on natural disasters or other forms of unintentional accidences. Hurricanes, for example, have been compared to a weapon of mass destruction (WMD) with complete critical infrastructure failure (Miller 2006) resulting from their impact. Critical Infrastructures were later defined by President Clinton’s Executive Order 13010 of 1996, establishing the President’s Commission on Critical Infrastructure Protection (PCCIP) with initial objectives to evaluate the scope and nature of threats and weaknesses of critical infrastructures (Executive Order No. 13010 1996). An excerpt from this executive order is as follows:

> Certain national infrastructures are so vital that their incapacity or destruction would have a debilitating impact on the defense or economic security of the United States. These critical infrastructures include telecommunications, electrical power systems, gas and oil storage and transportation, banking and finance, transportation, water supply systems, emergency services (including medical, police, fire, and rescue), and continuity of

> government. Threats to these critical infrastructures fall into two categories: physical threats to tangible property ("physical threats"), and threats of electronic, radio-frequency, or computer-based attacks on the information or communications components that control critical infrastructures ("cyber threats"). Because many of these critical infrastructures are owned and operated by the private sector, it is essential that the government and private sector work together to develop a strategy for protecting them and assuring their continued operation. (Executive Order No. 13010 1996)

For a history of critical infrastructure in the U.S. up through the PCCIP, see Brown (2006).

Rinaldi et al. (2001) describe the critical infrastructures of the U.S. as interdependent by physical means and also through information and communications systems. They describe ripple effects through direct and indirect failures within infrastructures. They use examples of the Galaxy 4 telecommunications satellite and California's prolonged power crisis to demonstrate how the direct and indirect failures in one infrastructure through interdependencies, cause failures in others. The conceptual framework provides our basis for work on interdependent systems. They define six dimensions of interrelated factors illustrated in Figure 1. They also touch on limitations and challenges of developing and validating models for infrastructure interdependency analysis, due to the complex of the relationship and difficulty of merging multiple systems into a single program.

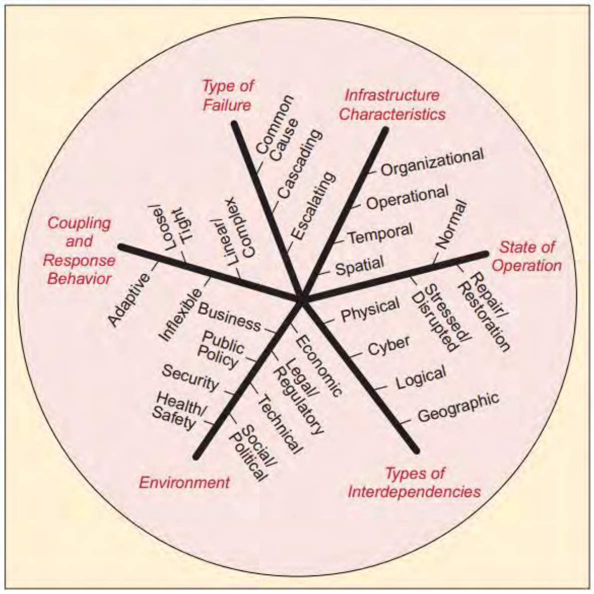

Figure 1. The six dimension perspective of infrastructure interdependency (from Rinaldi et al. 2001).

Lee et al. (2007) discusses interconnectivity of infrastructures, focusing on how failures in one system may lead to disruptions in another. Their work segregates the systems into separate mathematical models specifically identifying the interdependencies by type and impact. Five types of infrastructure interdependencies are presented and incorporated into a network flows mathematical representation. They include:

- Input dependence: An infrastructure that requires input commodity from one or more other infrastructures complete its service or function.
- Mutual dependence: Two or more infrastructures where each has a required input from the other to complete its service or function.
- Shared dependence: Some commodity flow from an infrastructure is required in completing the service or function of two or more infrastructures.
- Exclusive-or dependence: An infrastructure can only produce or supply one service, function, or commodity at a time and it can only be received by one infrastructure of two or more at a time.
- Collocated dependence: Part of two or more infrastructures are located in the same geographical location.

Lee et al. introduce three models of infrastructures from New York's lower Manhattan region, including realistic data on the interconnections of the power, telecommunications, and subways, and then synthesizes a scenario that causes major disruptions in all the services, and finally demonstrating the use of the model as a guide to restore lost services.

Kennedy et al. (2009) discusses how traditional infrastructure network models often lack the complexity to directly account for interdependent nature of critical infrastructures. His research describes a model of individual infrastructures together with special attention to modeling the interconnected dependencies. The model uses two sets of variables, one set that represents the infrastructure characteristics and another set that represents the specific interdependent elements. The mathematical formulation is then solved using Benders decomposition based on partitioning.

Dixon (2011) builds on the definitions and theory described by Lee et al. (2007) demonstrating how to build dependent relationships between separate models. He then incorporates worst case attack on the interdependent systems with the responding best use of available resources increase resiliency, on small scale infrastructure models. Solutions to Dixon's formulation show how dependent relationships between infrastructures can be used to explore vulnerabilities not available within the single-infrastructure models.

Gun (2013) considers the development of a cloud-based computational platform for modeling and analysis of interdependent infrastructure systems. In his implementation, independent operator models reside on a computational server, whose architecture supports model interconnection essentially by automatically taking output data from one model and passing it as input to another model. Gun develops a novel scripting language for defining these data dependencies and automating their execution, and he implements a proof-of-concept prototype server that demonstrates the effectiveness of the solution for a simplified example of interdependent infrastructure systems optimization. However, this work does not consider any exploration of new mathematics or algorithms for solving the operation with these interdependencies.

2. Electrical Transmission Network

In the IEEE Reliability Test System -1996, the Reliability Test System Task Force describes a reliability test system referred to as RTS-96, designed as a reference system to test reliability evaluation techniques, without any specific power system in mind. Figure 2 provides the structure of the RTS-96.

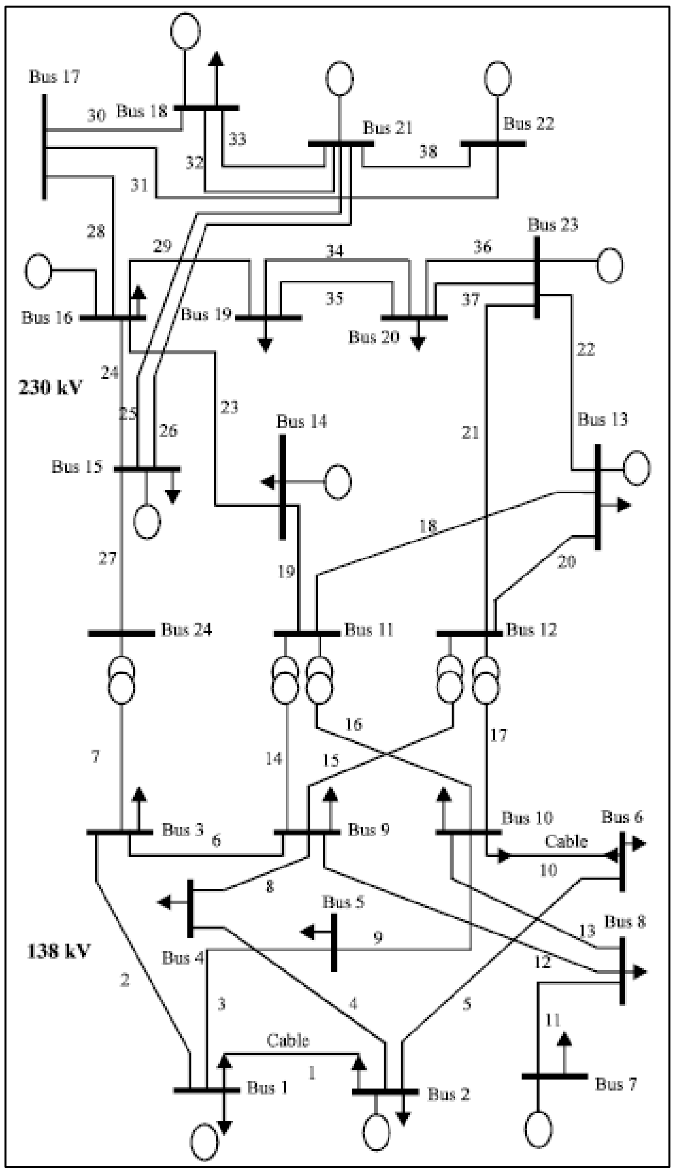

Figure 2. The IEEE One Area RTS – 96 is the basis for both Salmeron et al. (2004) and our electrical power system. It has 74 total nodes comprised of 33 generator nodes, 17 demand nodes, and 24 bus nodes. (from RTS Task Force 1999)

Salmerón et al. (2004) uses RTS-96 to create an electrical transmission model that closely approximates the actual physical characteristic and behaviors of power flow in a well-known and studied system. This is accomplished using a direct-current optimal power flow model, neglecting nonlinear losses and active power effects.

3. Fuel Transmission Network

Alderson et al. (2014) discuss the repeated disruptions in the critical infrastructures of the United States in recent years from various events, including natural disasters, accidental failures, and intentional attacks. Key to their discussion is the idea of using an operational model to assess system resilience to determine the best possible improvements to increase resilience. They demonstrate how their model determines optimal responses much the same way as real infrastructure owners and operators would. The scale and *N*-1 reliable design of the fuel system from Alderson et al. (2014) make it suitable for use as part of our research. Figure 3 illustrates the fuel network from Alderson et al. (2014).

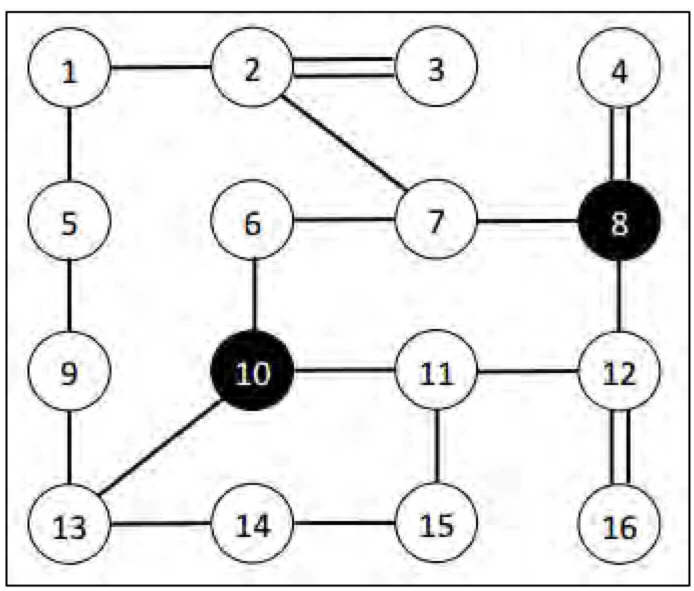

Figure 3. The network represents a fuel distribution model where the black filled circles are the supply nodes and the others have fuel demands, and the arcs represent establish connections between nodes (from Alderson et al. 2014).

B. OUR CONTRIBUTION IN CONTEXT

Our research is an extension of Dixon’s initial work to a case study on realistic and moderately sized networks. We look towards the practical application of modeling interdependent critical infrastructure systems to determine if small sets of attacks can have a systemic, disproportionate effect, in other words a small unrecognized set of components that have little effect on a system in isolation have a dramatic effect or cost on our objective function in an inter-dependent model. We also compare the results from maintaining separate models with interdependent connections solved iteratively to the whole interdependent set of infrastructures modeled as one monolithic network, and concurrent or when solving a sequence of single models yields result that are comparable to those derived from a monolithic model. We discuss situations in which an iterative algorithm that solves a sequence of single-infrastructure models converges to (or nearly to) the solution of a monolithic model that considers all infrastructures simultaneously.

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III. MODEL FORMULATION

Critical infrastructures are frequently modeled in isolation. If there are important interdependencies between two infrastructures, these separate models can be combined into a single monolithic model that accounts for their interdependencies explicitly. However, in many cases these separate models cannot be combined (for instance, the entities involved might not want to share operational details with each other), and so other solution methods must be used that maintain the separate models to the extent possible.

The dependent relationship described above is implemented in Dixon (2011). The graphical representation in Figure 4 shows a single dependence of one infrastructure on another.

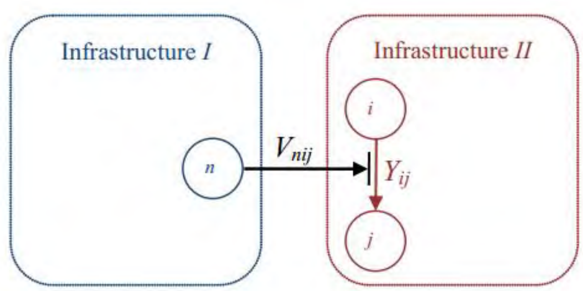

Figure 4. Graphical representation from Dixon (2011) showing the requirement of a commodity from infrastructure I by II.

In this figure, the activity Y_{ij} along arc (i,j) in Infrastructure II depends on the availability of a resource at node n, represented by V_{nij}, in infrastructure I. The mathematical form of the corresponding constraints follows as:

$$threshold_{ij} \cdot T_{ij} \leq V_{nij}$$

$$Y_{ij} \leq u_{ij} \cdot T_{ij}$$

where $threshold_{ij}$ is the minimum amount of commodity from one infrastructure (I) to allow a component of the second infrastructure (II) to function properly. V_{nij} is the flow of commodity from node n in infrastructure I to support the operation of component (i, j) in infrastructure II. Y_{ij} is the flow across component (i, j) in infrastructure II. T_{ij} is a binary variable that is used to determine if the threshold requirement is met. T_{ij} is set to zero when the flow V_{nij} of the required commodity is below the requirement $threshold_{ij}$ and may be set to one when V_{nij} is greater than or equal to $threshold_{ij}$. The parameter u_{ij} represents the capacity of flow from nodes *i* to *j*. Y_{ij} is the commodity flow variable from *i* to *j* in infrastructure II. Note the two commodity streams are not joined or mixed. If node *n* in infrastructure I represents a demand node for a required commodity of a component within infrastructure I, and if the demand V_{nij} is met in infrastructure I, then Y_{ij} in infrastructure II has capacity u_{ij} available.

In this thesis, the two infrastructure systems in question are an electric power system and a fuel distribution system. Associated with each system is an operational model of the system, sometimes referred to as an "Operator Problem." For the electric power system we have a Power Flow Model (PFM), and for the fuel system we have a Fuel Distribution Model (FDM). To distinguish between the models for these two systems, we adapt standard network flow notation. Specifically, we use a 'p' prefix to represent components of the electrical power network and an 'f' prefix to represent components of the fuel distribution model (see Figure 5).

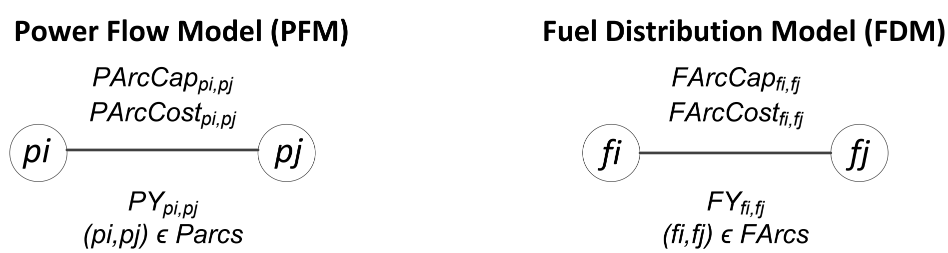

Figure 5. Distinctions in basic network flow notation for the electric power system and fuel distribution system.

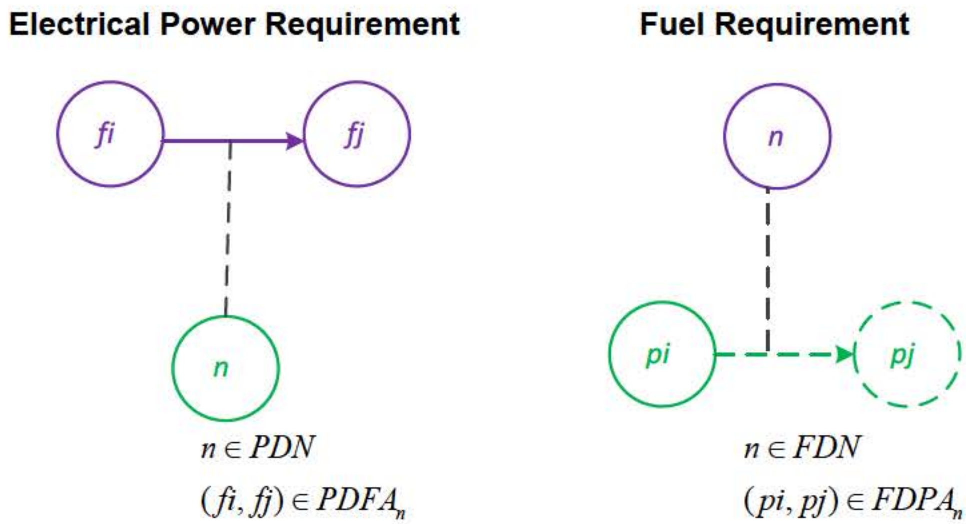

Figure 6. Convention for establishing interdependent sets between electric power and fuel distribution systems.

Our work centers around modifying existing isolated critical infrastructure models using the methods initially described by Dixon (2011) to account for inter-dependencies, and then solving to feasibility using an our iterative algorithm with the goal of determining if a small set of components within the combined systems may have an unusually drastic effect on the objective function. To demonstrate our ideas we start with two well-understood infrastructure networks with established, published, and proven formulations. The first is from Alderson et al. (2014), a FDM with their operator formulation. The other is the IEEE RTS-96 with the PFM from Salmerón et al. (2004).

To establish the inter-connections between the two infrastructures and represent dependencies, we first must understand how and why the dependencies exist. Here we have a fuel system that requires power to operate pumps, valves, heating elements, etc., and an electrical distribution system that has generators that require fuel. The generators are represented as power sources in the PFM and fuel demand nodes within the FDM. The fuel oil suppliers including big pumping stations are represented as fuel supply nodes, specified arcs within the FDM, and are represented as power demand nodes requiring electricity within the PFM.

The next two sections, III A & B, describe how we implement the adaptations to Dixon (2011). In section III C we describe the how we implement the large combined model including both the electric and fuel systems that, when solved, establish the optimal solution (i.e.,. setting the target). The final formulation presented in III D, is our subroutine used to communicate the reduced need for power capacity by the power system from the fuel system.

A. SETS, DATA, AND VARIABLES FOR OUR FORMULATIONS

We start by establishing indices and index sets used through our formulations.

1. Indices and Index Sets

$n \in FN$	nodes in fuel network (alias fi, fj)
$n \in FD \subseteq FN$	demand nodes in fuel network
$n \in FS \subseteq FN$	supply nodes in fuel network
$FArcs \subseteq FN \times FN$	arcs in fuel network
$n \in PN$	nodes in power network (alias pi, pj)
$n \in PD \subseteq PN$	demand nodes in power network
$n \in PG \subseteq PN$	generation (supply) nodes in power network
$n \in PI \subseteq PN$	bus nodes in power network (where supply = 0)
$PArcs \subseteq PN \times PN$	arcs in power network
$n \in PDN \subseteq PN$	power demand nodes that supply fuel components
$(fi, fj) \in PDFA_n \subseteq FArcs$	power-dependent fuel arcs: $(fi, fj) \in PDFA_n$ can only carry flow if power supply to $n \in PDN$ exceeds a given threshold.
$n \in FDN \subseteq FN$	fuel demand nodes that supply power components

$(pi, pj) \in FDPA_n \subseteq PArcs$ fuel-dependent power arcs: $(pi, pj) \in FDPA_n$ can only carry flow if fuel supply to $n \in FDN$ exceeds a given threshold

2. Parameters (Units)

$FSupply_n$	fuel supply at node $n \in FN$ [bbl/hr]
$FLdSCost_n$	fuel load shedding cost of demand node $n \in FD$ [$/bbl]
$FArcCost_{fi,fj}$	per-unit cost to move fuel on arc (fi, fj) [$/bbl]
$FArcCap_{fi,fj}$	capacity of fuel on arc (fi, fj) [bbl/hr]
$PDem_n$	power demand at node *n* [MW]
$PArcCap_{pi,pj}$	power capacity of (pi, pj) [MW]
$PY_{pi,pj}$	power flow across power arc (pi, pj) [MW]
$PThresh_n$	power threshold required by power demand node $n \in PDN$ [MW]
$PGenCap_n$	power generator capacity of $n \in PG$ [MW]
$PGenCost_n$	power generator cost per MW by node $n \in PG$ [$/MWh]
$PLdSCost_n$	power load shedding cost of node $n \in PD$ [$/MWh]
$PArcRes_{pi,pj}$	resistance of arc (pi, pj) $[\text{ohms}]$
$PArcRea_{pi,pj}$	reactance of arc (pi, pj) $[\text{ohms}]$
$B_{pi,pj}$	susceptance of arc (pi, pj) $[1/\text{ohms}]$
$FY_{fi,fj}$	fixed fuel flow across arc (fi, fj) [bbl/hr]
$FThresh_n$	fuel threshold required by power generation node $n \in FDN$ [bbl/hr]

3. Decision Variables (Units)

$FY_{fi,fj}$	flow on fuel arc (f_i, f_j) [bbl/hr]
$FLdS_n$	load shedding at fuel demand node $n \in FD$ [bbl/hr]
FT_n	$= \begin{cases} 1 & \text{if net supply to power node } n \in PDN \text{ meets or exceeds threshold} \\ 0 & \text{otherwise} \end{cases}$
$PGen_n$	power generated at generator node $n \in PG$ [MW]
$PY_{pi,pj}$	flow on power arc (pi, pj) [MW]
$PLdS_n$	load shedding at power demand node $n \in PD$ [MW]
θ_n	phase angle at power node $n \in PN$ $[\text{radians}]$
PT_n	$= \begin{cases} 1 & \text{if net supply to power node } n \in FDN \text{ meets or exceeds threshold} \\ 0 & \text{otherwise} \end{cases}$

B. FUEL DISTRIBUTION MODEL FORMULATION

This section presents details for our generalized interdependent operator's model of the fuel distribution model (FDM). This formulation is an expansion of concepts discussed in Chapter II as well as in Alderson et al. (2014). Without the loss of generality, the following mixed-integer formation represents a standard, single-commodity network flow problem with supplies and demands.

1. Formulation of Fuel System Operators Model

$$\min_{\substack{FY,FLdS\\FT}} \sum_{(fi,fj)\in FArcs} FArcCost_{fi,fj} FY_{fi,fj} + \sum_{n\in FD} FLdSCost_n FLdS_n \qquad \text{(F0)}$$

s.t.

$$\sum_{\substack{fj:(n,fj)\\ \in FArcs}} FY_{n,fj} - \sum_{\substack{fi:(fi,n)\\ \in FArcs}} FY_{fi,n} - FLdS_n \le FSupply_n \qquad \forall n\in FN \qquad \text{(F1)}$$

$$FY_{fi,fj} + FY_{fj,fi} \le FArcCap_{fi,fj} \qquad \forall (fi,fj)\in FArcs \qquad \text{(F2)}$$

$$FY_{fi,fj} + FY_{fj,fi} \le FArcCap_{fi,fj} FT_n \qquad \forall n\in PDN, (fi,fj)\in PDFA_n \qquad \text{(F3)}$$

$$PThresh_n FT_n \le \sum_{pi:(pi,n)\in PArcs} PY_{pi,n} - \sum_{pj:(n,pj)\in PArcs} PY_{n,pj} \qquad \forall n\in PDN \qquad \text{(F4)}$$

$$FY_{fi,fj} \ge 0 \qquad \forall (fi,fj)\in FArcs \qquad \text{(F5)}$$

$$FT_n \in \{0,1\} \qquad \forall n\in PDN \qquad \text{(F6)}$$

2. Discussion

The objective function, (F0), calculates the total cost of fuel transport across arcs in the network and the total penalty paid for unsatisfied demand. Constraints (F1) enforce balance of flow at each node. Constraints (F2) and (F5) ensure bounds on the fuel flow decision variables, $FY_{fi,fj}$. Constraints (F3) enforce bounds on flow decision variables when the dependence threshold variables are set to zero. Constraints (F4) set the dependence threshold variable FT_n based on the last known operating conditions in the power system. Constraints (F4) are the only place in which we model the power system, and although we could eliminate (F4) entirely by pre-calculating the values of the FT_n variables, we retain them because they exactly parallel one set of interdependence constraints from the monolithic formulation. Constraints (F6) require that dependence threshold variables, FT_n, are binary.

In this formulation we use two non-negative variables to solve for the flow paths of the commodity, FY_{ij}, and the amount of commodity shorted to customer n, $FLdS_n$. The last variable is a binary switch modeling the interdependence,

FT_n, which is set to one when the net supply received by demand node n within the electrical distribution system meets or excessed the power requirement or threshold, $FThresh_n$, allowing full capacity for the corresponding fuel path on arc (fi, fj). The model also only allows for positive flow and assumes a set contracted cost for service. The driving force for commodity flow is to avoid shortfall penalties for load shedding and minimizing delivery cost i.e., pumping path length, maximizing efficiency. The model features elastic parameters in the form of penalties for load shedding, maintaining feasibility and easy with which to work.

C. POWER FLOW MODEL FORMULATION

The following formulation describes our second generalized interdependent network, represented as a power flow model (PFM). This formulation is an expansion of concepts discussed in Chapter II from Salmerón et al. (2004) and Dixon (2011). Without the loss of generality, we build on the mixed-integer formulation using a direct-current approximation of an alternating-current electrical transmission system. We use the attributes that align with the actual RTS-96. The RTS-96 includes multiple types of power generation with associated costs per Megawatt-hour (MWhr).

1. Formulation of Power Flow Model with Interdependence

$$\min_{\substack{PY,PGen\\PLdS}} \sum_{n\in PG} PGenCost_n PGen_n + \sum_{n\in PD} PLdSCost_n PLdS_n \qquad \text{(P0)}$$

s.t.

$$PY_{pi,pj} = B_{pi,pj}\left(\theta_{pj} - \theta_{pi}\right) \qquad \forall (pi,pj)\in PArcs \qquad \text{(P1)}$$

$$\sum_{\substack{pi:(pi,n)\\ \in PArcs}} PY_{pi,n} - \sum_{\substack{pj:(n,pj)\\ \in PArcs}} PY_{n,pj} = 0 \qquad \forall n\in PI \qquad \text{(P2)}$$

$$PGen_n + \sum_{\substack{pi:(pi,n)\\ \in PArcs}} PY_{pi,n} - \sum_{\substack{pj:(n,pj)\\ \in PArcs}} PY_{n,pj} = 0 \qquad \forall n\in PG \qquad \text{(P3)}$$

$$\sum_{\substack{pi:(pi,n)\\ \in PArcs}} PY_{pi,n} - \sum_{\substack{pj:(n,pj)\\ \in PArcs}} PY_{n,pj} + PLdS_n = Pdem_n \qquad \forall n\in PD \qquad \text{(P4)}$$

$$FThresh_n PT_n \le \sum_{\substack{fi:(fi,n)\\ \in FDPN}} FY_{fi,n} - \sum_{\substack{fj:(n,fj)\\ \in FDPN}} FY_{n,fj} \qquad \forall n\in FDN \qquad \text{(P5)}$$

$$PY_{pi,pj} \le PArcCap_{pi,pj} PT_n \qquad \forall n\in FDN, (pi,pj)\in FDPA_n \qquad \text{(P6)}$$

$$0 \le PY_{pi,pj} \le PArcCap_{pi,pj} \qquad \forall (pi,pj)\in PArcs \qquad \text{(P7)}$$

$$0 \le PGen_n \le PGenCap_n \qquad \forall n\in PG \qquad \text{(P8)}$$

$$0 \le PLdS_n \le PDem_n \qquad \forall n\in PD \qquad \text{(P9)}$$

$$PT_n \in \{0,1\} \qquad \forall n\in FDN \qquad \text{(P10)}$$

2. Discussion

The objective function, (P0), calculates the total cost to generate power and the total penalty cost of load shed (i.e., unmet demand). Constraints (P1) approximate active power flows on the arcs. Constraints (P2) maintain power balance at bus nodes. Constraints (P3) maintain power balance at generator nodes. Constraints (P4) maintain power balance at demand nodes. Constraints (P5) set the dependence threshold variable PT_n based on last know operating conditions in the FDM. Constraints (P5) are the only place where the fuel system is considered, and while we could pre-calculate the values of PT_n variables, we retain them because they exactly parallel one set of interdependence constraints from the monolithic formulation. Constraints (P6) enforce restricted bounds on power flow decision variables when the dependence threshold variable is set to zero. In this formulation we use three continuous variables to solve for the flow paths of the electrical power, $PY_{pi,pj}$, the amount of electrical power shorted to customer n, $PLdS_n$, and the amount of electrical power supplied by generator n, $PGen_n$, The last variable is a binary switch, PT_n, which is set to one when net fuel supplied to demand node n in the FDM meets or exceeds its fuel requirement (threshold), $FThresh_n$. This allows full capacity for the corresponding interdependent power components. It is understood that the path of electrical power follows Kirchhoff's Law, but computation is required within our model to complete power balances at each node, not for determining optimal flow patterns of electricity. The power flow model differs from the FDM in that it has assigned costs for generating and shedding electrical power, and does not include a transmission costs. The driving force for power generation and flow to demand nodes is to minimize the objective function where it is more efficient to generate power using the cheapest generators available and transport it to the demand nodes, than to pay the load shedding cost for load shedding.

D. FULL COMBINED MODEL

The following formulation describes the combination of the FDM and Power Flow models. This formulation includes fuel supply and demand nodes, power supply and demand nodes as well as constraints requiring flow balances at each node which correspond to the specific type of node (bus, generator, demand). It also explicitly models the interdependencies between the two systems.

1. Formulation of the Full Combined Model

$$\min_{\substack{FY,FLdS\\FT,PY,PY\\PGen,PLdS}} \sum_{(fi,fj)\in FArcs} FArcCost_{fi,fj} FY_{fi,fj} + \sum_{n\in FD} FLdSCost_n FLdS_n$$

$$+\sum_{n\in PG} PGenCost_n PGen_n + \sum_{n\in PD} PLdSCost_n PLdS_n \qquad (\mathrm{C0})$$

s.t.

$$(\mathrm{F1}),(\mathrm{F2}),(\mathrm{F3}),(\mathrm{F5}),(\mathrm{F6})$$

$$(\mathrm{P1}),(\mathrm{P2}),(\mathrm{P3}),(\mathrm{P4}),(\mathrm{P6}),(\mathrm{P7}),(\mathrm{P8}),(\mathrm{P9}),(\mathrm{P10})$$

$$PThresh_n FT_n \le \sum_{\substack{pi:(pi,n)\\ \in PArcs}} PY_{pi,n} - \sum_{\substack{pj:(n,pj)\\ \in PArcs}} PY_{n,pj} \qquad \forall n \in PDN \qquad (\mathrm{C1})$$

$$FThresh_n PT_n \le \sum_{\substack{fi:(fi,n)\\ \in FArcs}} FY_{fi,n} - \sum_{\substack{fj:(n,fj)\\ \in FArcs}} FY_{n,fj} \qquad \forall n \in FDN \qquad (\mathrm{C2})$$

2. Discussion

The objective function (C0) simply calculates the sum of the costs in each system as presented in (F0) and (P0). Constraints (C1) and (C2) model the interdependencies between the two systems. Constraints (C1) require that net supply of electrical power to power demand nodes meet or exceed the threshold, $PThresh_n$, to allow fuel flow on the corresponding FDM arc (fi, fj). Constraints (C2) require the net supply of fuel to fuel demand node n meet or exceed the threshold requirement, $FThresh_n$, to allow flow on the corresponding interdependent power arc (pi, pj).

E. EXCESS CAPACITY SUBPROBLEM

The following formulation is an expansion of the standard knapsack problem. If there is excess power generation capacity (i.e., more power is being generated than is needed by all of the demands in the system, including interdependent demand from the fuel system), it determines which dependent generators from the set, *GRF*, could be shut down without impacting the power supplied to the demand nodes while saving fuel transportation costs.

1. Indices and Index Sets:

$n \in PG \subseteq PN$	power generation nodes in the power networks
$n \in PD \subseteq PN$	power demand nodes in the power networks
$n \in GRF \subseteq PN$	power generation nodes that require fuel from FDM

2. Parameters (Units):

$PDem_d$	power demand of node *d* $[\text{megawatts}]$
$PGenCap_g$	power generator capacity of *g* $[\text{megawatts}]$
$PGenCost_g$	power generator cost per MW by node *g* $[\text{dollars/megawatt} \cdot \text{hours}]$

3. Decision Variables (Units):

OFF_n = $\left\{ \begin{array}{ll} 1 & \text{if generator } n \text{ is made unavialable} \\ 0 & \text{otherwise} \end{array} \right\}$

4. Formulation of the Subroutine:

$$\max_{OFF} \sum_{n \in GRF} PGenCost_n PGenCap_n OFF_n \qquad (\text{K0})$$

$$s.t. \quad \sum_{n \in GRF} PGen_n OFF_n \le \sum_{n \in PG} PGenCap_n - \sum_{n \in PD} PDem_n \qquad (\text{K1})$$

$$OFF_n \in \{0,1\} \qquad \forall n \in GRF \qquad (\text{K2})$$

5. Discussion

The objective function, (K0), determines the cost savings that can be realized by turning off a subset of excess oil dependent generators. The decision variables are a matrix of binary variables, OFF_n, one for each fuel oil dependent generator. If OFF_n is equal to 1 then generator n is not used, and the corresponding fuel demand in FDM is set to 0. If OFF_n is set to 0 then generator n is available and corresponding demand in the FDM exists. This model is used to remove excess generator capacity so as to reduce the fuel requirements and create a solution closer to the optimal in the overcapacity case.

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IV. ANALYSIS AND RESULTS

We expand on the ideas from Lee et al. (2007) and Dixon (2011) to develop a realistic and reasonably-sized instance of two interdependent infrastructure systems. We solve the resulting operational problem with our monolithic combined model to establish a baseline optimal solution, and then apply our iterative solution algorithm using the two separate models, using a few different policies to set the penalties on unmet demand in the dependent infrastructure components. We comment on how these different penalty policies can affect the solution reached by our algorithm, and how they can be used to influence the performance of the combined system. Finally, we implement a knapsack subproblem that we use as part of a subroutine in our algorithm to help eliminate wasteful fuel distribution. This routine helps close the optimality gap in cases where we have excess capacity in one or both systems.

A. THE CASE STUDY BACKGROUND AND SETUP

To demonstrate our methods on dependent infrastructures, we use two previously published systems that give us the ability to understand and anticipate results as well as help verify and validate our formulations. While we focus on a specific example to demonstrate our methods, we do so without loss of generality; our models and algorithms scale with the size of the individual infrastructure models involved.

The RTS-96 system (Figure 2) is an electrical generation and transmission system. The RTS-96 has 33 generator nodes with various generation types, including nuclear, coal, oil, and hydroelectric, all with different associated costs. It has 17 different demand nodes with different requirements, and 24 bus nodes that connect the power generation to power demands in a realistic way. Salmeron et al. (2004) provides a mathematical programming formulation of the operation of the RTS-96.

The fuel delivery system we model (Figure 3) is an *N*-1 reliable network having two supply nodes, 14 demand nodes, and bidirectional arc flows. The basis for modeling this system is a standard single commodity flow model with supplies and demands, minimizing total delivery costs, modified to add dependence on a separate infrastructure. The model has built-in elasticity with the possibility of shedding demand loading with an assigned penalty and is discussed in detail in Alderson et al. (2014).

Each infrastructure system model has dependencies on the other system: some of the fuel pumps in the fuel distribution model (FDM) require power from the electrical system, and some of the generators in the electrical system require fuel from the FDM.

The overall problem we wish to solve is to determine the optimal operation of both systems, so that all requirements (fuel and power) are met at minimum total cost. However, we anticipate that in a real setting we might not have access to models of both systems simultaneously, and that formulating and solving a single, monolithic model of the combined system might not be possible.

To this end, our operational model of each system represents the requirements of the other system as demands, with corresponding penalties for load shedding. The appropriate setting of these penalties for load shedding is key to the effective use of these models.

Our background scenario for inter-dependence follows concepts described by Lee et al. (2007) and demonstrated using small (e.g., three-node) networks in Dixon (2011). Although in reality inter-dependence often extends beyond just two networks, our system remains moderate in complexity, adding features which smaller instances, like those presented in Dixon (2011), could not. The commodity in the fuel delivery system is assumed to be number 6 fuel oil. We acknowledge the known difficulties and limitations associated with a fuel distribution model pumping this type of fuel due to its high viscosity at standard temperature, but assume those limitation have been overcome. RTS-96 was designed with prescribed power generation and demand loadings. These numbers are found in Figures 7 and 8 and are the basis for the fuel demand numbers within the FDM, applying elementary energy and unit conversions (Reliability Test System Task Force 1999).

From the U.S. Energy Information Administration (EIA) (Energy Information Administration 2014) we ascertain that about 533 kilowatt hours can be produced from a 42 gallon barrel of number 6 fuel oil. When operated by major U.S. investor-owned electric utilities, average power generation expenses in 2012 were as follows:

- Fossil stream: $31.89 per megawatt hour
- Nuclear: $25.48 per megawatt hour
- Hydroelectric: $11.34 per megawatt hour
- Small turbine: $35.69 per megawatt hour.

These prices are used as cost coefficients in our model for the corresponding power generating systems. Additionally, from the EIA, we find the current and projected cost per barrel of number 6 fuel oil is approximately $100; the penalty for load shedding is set at $150 per barrel in the FDM model for our base case.

To integrate the two systems and ensure that the dependencies remain within the same order of magnitude, we use EIA data starting with:

- the estimate of 533.4 kilowatt hours generated per barrel of number 6 fuel oil, and
- the RTS-96 capacity for the oil fired generators to calculate a baseline demand for fuel by each generator within the FDM.

Specifically, RTS-96 has 15 generators powered using number 6 fuel oil at five separate locations. Thus, we assume that the five separate locations are different fuel oil power plants fed by the FDM. This demonstrates the first dependent relationship between the two systems: an oil-fired generator node from the RTS-96 requires fuel supplies from the FDM in order to run. Table 1 shows which fuel demand nodes correspond to the specific fuel oil generators.

For example, generator node $g101c$ requires fuel from the FDM and is part of the fuel demand at node $fn15$.

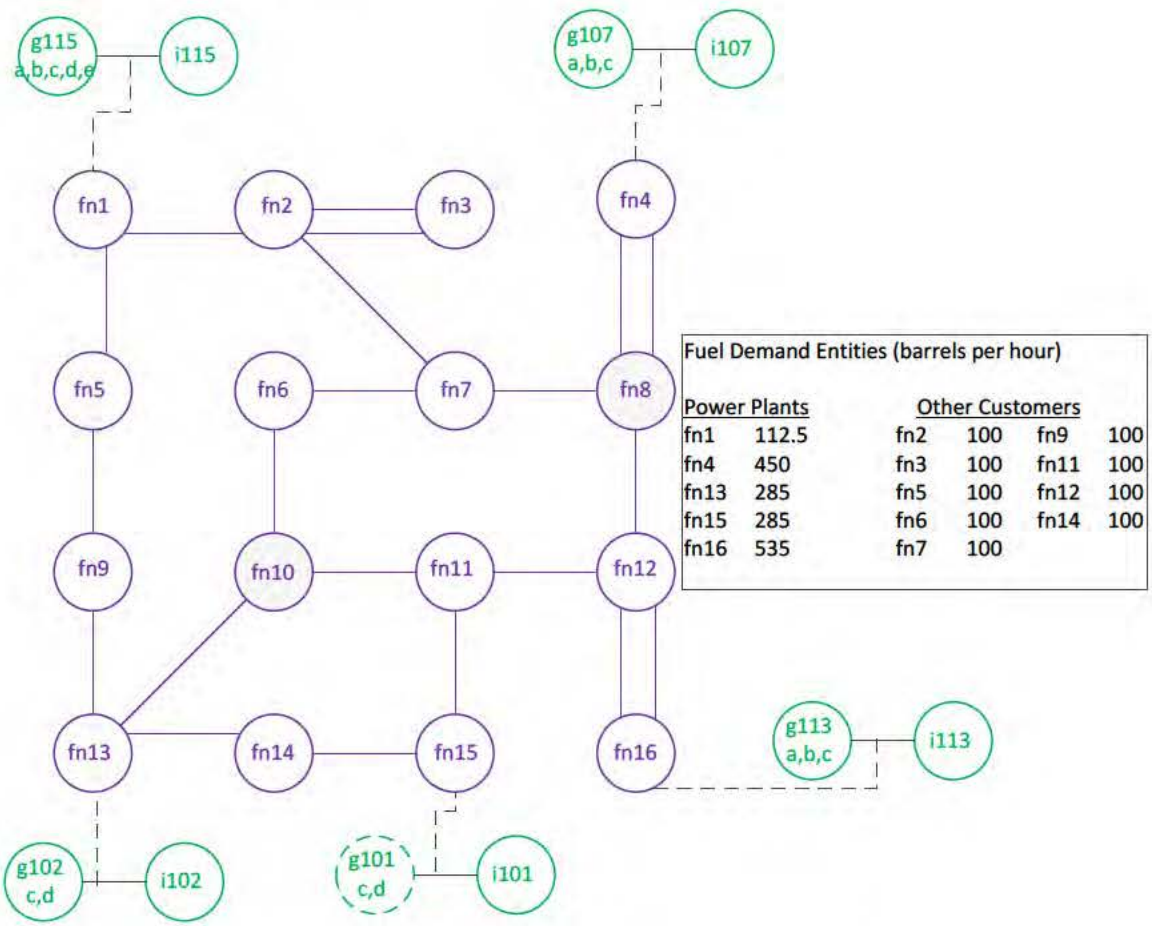
Oil Firer Generator nodes within RTS-96	Corresponding FDM Demand node	Generating Capacity (megawatt hours)	Fuel Demand (barrels / hr)
g101c,d	fn15	152	285
g102c,d	fn13	152	285
g107a,b,c	fn4	240	450
g113a,b,c	fn16	285.3	535
g115a,b,c,d,e	fn1	60	112.5

Table 1. Fuel dependent generators from RTS-96 and their corresponding nodes within the FDM with associate generating capacity and tabulated fuel demand from our calculations.

To model complex infrastructures with dependencies we opt to build and test the infrastructures as independent standalone networks, validate their responses against current knowledge or literature, then add inter-dependencies. This alleviates the requirement for one individual to learn about and construct models for every system with interdependence, and instead allows for the separate models to be built by different teams and then tied together and solved using algorithms like ours. With this process in mind we model the FDM and RTS-96 separately. The results for the FDM system were validated against the results in Alderson et al. (2014). The results for the IEEE RTS-96 system were similarly inspected for consistency and correctness: cheaper power generation is used before more expensive, all power flows balance, all demands are met or loads are explicitly shed when necessary, and production never exceeds demand.

We show how to incorporate two or more functioning models to include dependencies. In our simplified case study, the FDM requires electrical power to both heat and pump fuel from supply to demand nodes. While the *cost* of these functions is easily incorporated into the model without modeling the explicit dependence on the power model, there is no opportunity for modeling the consequences of losing this power. The same argument exists with respect to fuel required for the generators within RTS-96. No power can be generated without fuel, so modeling an electrical transmission system without an understanding of the connected FDM also eliminates our ability to calculate the consequence of losing this fuel supply. Figures 7 and 8 show the specific locations of dependence.

In Figure 7, we show the complete FDM in purple and interdependent generators of the PFM in green. The five power plants from RTS-96 are shown with all the associated generator nodes. The generator nodes require a fuel supply. The dashed lines show conceptually where the fuel supply manifests itself in the FDM as a demand node. For example, $g107a$ has fuel demand node $fn4$ in the FDM, and this demand must be filled for $g107a$ to produce power up to its capacity in the PFM.



Fuel Demand Entities (barrels per hour)

Power Plants		Other Customers			
fn1	112.5	fn2	100	fn9	100
fn4	450	fn3	100	fn11	100
fn13	285	fn5	100	fn12	100
fn15	285	fn6	100	fn14	100
fn16	535	fn7	100		

Figure 7. RTS-96 positions of fuel dependence

In Figure 8, we illustrate how the FDM receives electrical power from the PFM and which components specifically require electrical power to function. For example, the pumps and heating elements in arcs $(fn6, fn10)$ have a corresponding electrical demand node ($d103$) in the PFM. A total of four demand nodes from RTS-96 (in green) have interdependencies that are shown with the dashed line connecting it to its electrical component in the FDM.

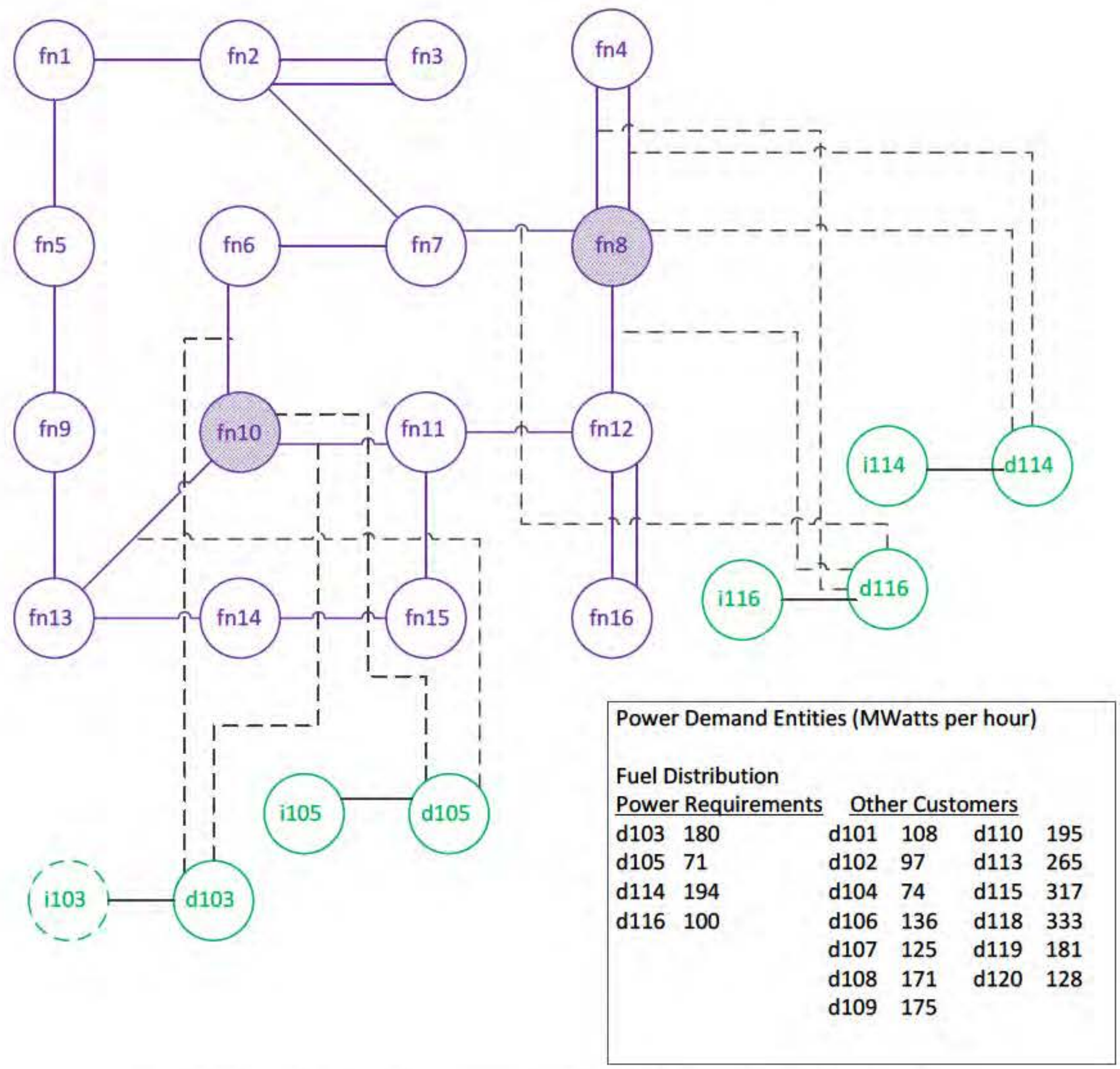


Figure 8. FDM positions of electrical power dependency

In summary, Figure 7 and Figure 8 present the supplies and demands for each system, along with the corresponding interdependencies *as viewed by each infrastructure in isolation*. However, as we observe in the next section, this view of the requirements imposed by supplies and demands changes when we consider the combined interdependent operation of both systems.

B. SOLUTION FROM THE COMBINED MODEL

The combined model solves for the optimal operation of both the electric power and fuel systems as an integrated whole. That is, the combined model seeks to achieve the lowest combined operating cost, with tradeoffs between the costs borne by each system defined by the coefficients in equation (C0). This is the solution that could be obtained if both infrastructure systems were operated by a single entity that has complete visibility of the internal details of both systems.

We use CPLEX 12.6.0 (GAMS 2014) with GAMS 24.2.1 (GAMS 2014) to solve the combined model as well as the individual models. The combined model consists of 1015 constraints over 606 variables, 38 of which are binary. CPLEX solves the combined model to an approximate 1% optimality gap within less than a tenth of a second. Solving the combined model yielded, in every attempt, a feasible and optimal solution for the overall objective function. The optimal solution to this monolithic formulation of our two systems together provides the optimistic lower bound we strive to attain using our algorithms for solving the interdependent operation using separate models for each system.

The combined model finds solutions that balance load shedding penalties in both systems and determines an operational plan that might not be obvious when modeling either system individually. For example, the combined model reveals inefficiencies associated with delivering fuel to a power plant that is not used except to provide power to the fuel system that feeds that power plant. When the power and fuel models are solved simultaneously, the combined model shuts off some of the generators as well as some of the fuel components to reduce the total operating cost. Specifically the combined model sheds the following internal electrical loadings:

- d103, (180mwhr)
- d114, (194mwhr)
- d116, (100mwhr)

Shedding these loads increases the objective value of the fuel portion of the system by forcing a portion of the fuel to travel longer distances along arcs (*fn6,fn10*), (*fn10,fn11*), (*fn4,fn8*), and (*fn8,fn12*). In addition, supply from supply node *fn8* can no longer be used without the corresponding electrical power demand being filled. We also observe a reduced demand within the fuel system, as the following fuel demands are eliminated with less power demanded:

- FN4, (450 barrels/hr)
- FN15, (285 barrels/hr)
- FN16, (535 barrels/hr)

Figure 9 shows the configuration of the nodes of the optimal solution for the combined model. We consider this solution and its objective value of $71,691, the best possible given the conditions. As we explore alternative methods to solve this problem maintaining separate models, we will use the results here as a baseline for comparison.

Fuel Node	Supply Demand
FN1	112.5
FN2	100
FN3	100
FN4	0
FN5	100
FN6	100
FN7	100
FN8	0
FN9	100
FN10	1297.5
FN11	100
FN12	100
FN13	285
FN14	100
FN15	0
FN16	0

Generator Node	Gen Cap
g101a	10
g101b	10
g101c	x
g101d	x
g102a	10
g102b	10
g102c	76
g102d	76
g107a	x
g107b	x
g107c	x
g113a	x
g113b	x
g113c	x
g114	0
g115a	12
g115b	12
g115c	12
g115d	12
g115e	12
g115f	155
g116	155
g118	400
g121	400
g122a	50
g122b	50
g122c	50
g122d	50
g122e	50
g122f	50
g123a	155
g123b	155
g123c	350

Demand Node	Load
d101	108
d102	97
d103	0
d104	74
d105	71
d106	136
d107	125
d108	171
d109	175
d110	195
d113	265
d114	0
d115	317
d116	0
d118	333
d119	181
d120	74

Interdependent Node
Shut down to minimize cost
Independent Loads Shed
x - Shed Forced by dependence

Figure 9. Results from the solving the combined (monolithic) model

C. ITERATIVE ALGORITHM

An optimal solution to our monolithic combined model provides the operations of both the fuel and power systems that minimize the total operating cost across both systems. We now use an iterative algorithm, as described in Figure 10, to alternate between solving the fuel model (with fixed power demands) and the power model (with fixed fuel demands), at each step passing enough of the solution to the current model as input data for the next model, until neither model's optional solution changes.

1) Determine feasible starting solution for fuel and power models

2) Repeat:

    a. Solve fuel model given fixed demands from current power solution
    b. Record current fuel solution and power requirement
    c. Solve power model given fixed demands from current fuel solution
    d. Record current power solution and its fuel requirements

3) Until fuel solution and power solution do not change

Figure 10. Pseudocode of iterative algorithm to solve for interdependent operation of fuel and electric power systems.

To solve for the operation of the two interconnected but separate models, our algorithm simply solves the fuel model, making the resulting flows available to the power model. Then it solves the power model making the results available to the fuel model. This repeats until the individual objective functions have converged. In this way, our iterative algorithm produces behavior that is similar to what might happen in practice if each system is operated independently, with information about demands coming only at the boundary between systems. It is possible that this interaction results in complicated transient behavior (e.g., oscillations or cycling), however, our focus is on the equilibrium solution of this interaction.

The fuel model consists of 161 constraints over 91 variables, 9 of which are binary. The power model consists of 869 constraints over 544 variables, 29 of which are binary. CPLEX solves each model to within a 1% optimality gap almost instantly, although the solution created by combining these individual infrastructure solutions is usually not within the same optimality gap for the combined model. We explore modifications to attempt to close the gap between the actual optimal solutions from the combined model and solving the separate models, ending with our knapsack type subroutine.

Using our iterative algorithm, we investigate the behavior of these interdependent infrastructure systems for two basic scenarios:

- Scenario 1: All generators available, such that power generation capacity is greater than demand;
- Scenario 2: Loss of 13.3% of generating capacity, resulting in power generation capacity less than demand.

For each of these scenarios, we consider the interdependent operation of these systems under three different operating conditions, defined by different penalty costs:

- Equal penalties on load shedding across all demand nodes in both models;
- Double the penalties on interdependent demand nodes than that of independent (i.e., customer) nodes for load shedding;
- No penalties on interdependent demand nodes for load shedding.

The following sections discuss the results for these scenarios using our iterative algorithm, accompanied by analysis and insights.

D. SCENARIO 1: SYSTEM OPERATION WHEN THERE IS EXCESS POWER GENERATION CAPACITY

We now describe the results from our iterative algorithm in the scenario when there is an excess power generating capacity.

1. Equal/constant load shedding penalty on all demand nodes from each system.

In the base case for our analysis, we assume that all generators are available (so there is excess generation capacity) and the load shedding penalties are high and equal for all demand nodes across both systems.

Table 2 summarizes the progression of our algorithm for the base case when solving interdependent operator models. In iteration 1 we solve the FDM model to optimality as an isolated system. We observe that all the fuel demands are met in this iteration. The results are then passed as parameters to the PFM. In iteration 2 the PFM is solved to optimality with the results passed as parameters back to the FDM. Solving our base case requires only one solve per model, or two total iterations, in order to achieve stable results. The sum of the FDM and PFM objective values is the total objective value and in this configuration is $84,053, which is the best obtainable value ignoring interdependence and assuming each system is isolated. Comparing this to the solution from the combined model we see the iterative algorithm yields results that are 17.2% higher. The operating solution obtained when using our iterative algorithm for the separate models results in fuel being supplied to every demand node. Thus, we observe that even though each system is performing optimally in isolation, the overall operating cost is higher than it could be if the two systems were operated as a single entity. This can be explained looking closer at the combined model results.

The FDM objective value increases by 6.2% when solved as a combined model which would not happen in isolation no matter how many iterations were run. The increase comes from shedding fuel demands which are providing fuel to an over capacity electrical power generation system at a cost higher than its return value. The PFM objective value goes down in response, due to less electricity demand from the fuel model due to shutting down nonessential fuel model components, reducing it by 15.9% versus the PFM being solved in isolation. Thus, we see a net decrease of 14.7% for the entire system.

Full Generating Capacity Available & Constant and Equal Load Shedding Penalty on all demand nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	79593	84053	Optimal PFM solution in isolation, no load shedding
3	FDM	4460	79593	84053	No Change, stable condition met in FDM
4	PFM	4460	79593	84053	No Change, stable condition met in PFM
Combined Model		4735	66956	71691	Combined solution has total costs 17.2% lower than iterative solution

Table 2. Results from our initial iterative algorithm with all generators available and a constant and equal load shedding penalties on all demand nodes for each system.

2. Double the load shedding penalty on all the interdependent demand nodes for each system

In this case, we increase the load shedding penalty costs for the interdependent demand notes by a factor of two. We expect this will make each standalone system more sensitive to a potential shortfall in the other, and therefore not likely to short those demands, choosing instead to short demands on its customers first. Table 3 summarizes the progression of our iterative algorithm for this case. Iteration 1 of our algorithm solves the FDM model to optimality as an isolated system. We observe that all the fuel demands were met, and the resulting solution is passed as parameters to the PFM. Iteration 2 solves the PFM to optimality; in this solution all electrical demands are met using the minimum cost set of generators. The results are passed as parameters back to the FDM in Iteration 3 where we find the stable solution. The total objective value in this configuration is $84,053, as before. As expected, the higher load shedding penalties (and excess commodity capacities) drove the separate models to supply commodity to every demand node.

Full Generating Capacity Available & Double Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	79593	84053	Optimal PFM solution in isolation, no load shedding
3	FDM	4460	79593	84053	No Change, stable condition met in FDM
4	PFM	4460	79593	84053	No Change, stable condition met in PFM
Combined Model		4735	66956	71691	The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr.

Table 3. Results from our initial iterative algorithm with all generators available and double the load shedding penalties on interdependent demand nodes. * Independent demand node.

3. No load shedding penalty on all the interdependent demand nodes for each system.

To continue exploring the effects of penalties on the interdependent nodes, we now alter our base case by making the load shedding penalties on interdependent demand nodes zero. Table 4 summarizes how the algorithm progresses. Iteration 1 of our algorithm solves the FDM model to optimality as an isolated system. Here we observe that the lack of a load shedding penalty on the interdependent nodes results in a decision not to fill their associated fuel demand. The operating cost for the FDM is cut by 64% when it does not deliver fuel to these nodes. The solution is then passed as parameters to the PFM. Iteration 2 solves the PFM to optimality as an isolated system. Because there are no load shedding penalties, we observe that all generators that require fuel oil to produce power (interdependent generators) do not produce any electricity—as a result the interdependent power demand nodes also do not receive power. The PFM uses its previously extra capacity with alternate and more expensive generators to meet power demands; this drives up the PFM objective value. But without the fuel oil generators there is not enough capacity to fill all the independent demands. Thus, we see load shedding from some of the independent demand nodes (in this case, d119 and d120 shed 195 megawatt hours of their power demand). The overall effect on the PFM objective value is a reduction of 11.1% to $70,771. The results are passed as parameters to the FDM in iteration 3. Iteration 3 solves the FDM again and all the interdependent fuel demand components requiring power do not receive any. This results in an entire FDM system shutdown, and the FDM objective value at its max of $135,000.

Comparing these results to our base case we see here an extra iteration is required, and the total objective value and in this configuration is $205,771 (187% from our combined model). We see the separate models behaving much differently than in our base case and in the combined model. In the optimal solution to the combined model balances the penalties for not filling dependent demands with those of not filling the demands of customers, and can determine exactly how much power and fuel to deliver to both. In the previous cases, the penalty for load shedding on the interdependent demands is the driving force to fill those demands, but having a zero load shedding penalty on dependent nodes ensures that no materials are delivered to them, which results in reduced capacity, which in turn results in higher load shedding penalties paid in the dependent system.

Full Generating Capacity Available & No Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	1600	N/A	N/A	All interdependent fuel demand nodes loadings were shed
2	PFM	1600	70771	72371	All interdependent power demand nodes loadings were shed plus d119-181 mwhr* & d120-14 mwhr*
3	FDM	135000	70771	205771	No fuel is delivered, all fuel demands are shed
4	PFM	135000	70771	205771	No Change, stable condition met in PFM
5	FDM	135000	70771	205771	No Change, stable condition met in FDM
Combined Model		4735	66956	71691	The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr.

Table 4. Results from our initial iterative algorithm with all generators available and no load shedding penalties on interdependent demand nodes. * Independent demand node.

E. SCENARIO 2: SYSTEM OPERATION WHEN GENERATING CAPACITY IS LESS THAN DEMAND

We now revisit the previous three cases for interdependent different penalty costs, in the scenario when there’s a failure to a major component, specifically a nuclear power plant, at node *g118*, with a 400 megawatt capacity. With the loss of this component the system total generation capacity is less than the total demand in the PFM.

1. Equal/constant load shedding penalty on all demand nodes from each system.

As before, we start with the interdependent penalties equal on all demand nodes within each system. Table 5 summarizes the progression of our algorithm for this scenario. Iteration 1 solves the FDM model to optimality as an isolated system. Like the base case, no loads are shed within the FDM. The results are passed as parameters to the PFM. The PFM is then solved to optimality in Iteration 2. With generating capacity less than power demand, we observe load shedding from some of the demand nodes. Because the demand nodes have equal penalties across each system, power could be shed from any of the demands or even from any subset of the group. If the power is shed from one or more of the interdependent demand nodes, then the effects would cascade to the fuel system. Recall from the combined model results, the optimal solution involves shedding both interdependent and independent demand nodes to reach the combined optimal solution. In the case of equal load shedding penalties, CPLEX solves with the following nodes and loads shed:

- d107 with 55.7 mwhr (independent demand node),
- d110 with 195 mwhr (independent demand node).

The results are passed as parameters to the FDM, but at this point the system has achieved a stable solution. The sum of the FDM and PFM objective values in this configuration is $97,576.

We resolve the combined model under the assumed loss of *g118* resulting in the total optimal objective value of $86,126 with the following nodes and loads shed:

- d103 with 180 mwhr (interdependent),
- d109 with 70.7 mwhr(independent).

Comparing the separate model solution to the combined model we see the separate model is 13.3% from optimality using our iterative algorithm. Comparing the combined model to this run of our algorithm, we continue to see a need to develop a dynamic way of establishing interdependent penalties.

Reduced Generating Capacity Available (g118 offline) & Constant Load Shedding Penalty on all demand nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	93116	97576	The following loads were shed: d107-55.7mwhr*, d110-195mwhr*.
3	FDM	4460	93116	97576	No Change, stable condition met in FDM
4	PFM	4460	93116	97576	No Change, stable condition met in PFM
Combined Model		6510	79616	86126	The following loads were shed: d103-180mwhr, d109-70.7mwhr*.

Table 5. Results from our initial iterative algorithm with reduced power generating capacity available (generator node g118 offline) and a constant and equal load shedding penalties on all demand nodes for each system. * Independent demand node.

2. Double the load shedding penalty on all the interdependent demand nodes for each system

Here again, we increase the load shedding penalties on interdependent nodes by a factor of two, with the reduced generating capacity. Table 6 summarizes the progression of our algorithm. Iteration 1 solves the FDM model to optimality as an isolated system. As before, we observe all fuel demands are met. The results are passed to the PFM as parameters. The PFM is then solved to optimality in Iteration 2. We see the same response as the previous run with equal and constant penalties across each system. The difference is that with the higher penalties on the interdependent nodes all independent demand node loading is shed before any of the interdependent demand loading. This case protects the integrity of the combined system before exogenous demands. Loads are shed first within the set of independent nodes, then in the interdependent demand node loads. In this case, the following nodes and loads are shed:

- d107 with 55.7 mwhr (independent demand node),
- d110 with 195 mwhr (independent demand node).

The results are passed as parameters to the FDM, after which the solution is determined to be stable. The total objective value in this configuration is $97,576. Comparing the solution for the separate model solution to that for the combined model, we see the separate model has a solution that is 13.3% from optimality using our algorithm. This case demonstrates how setting the penalty for interdependent demand nodes higher than the independent demand nodes improves the integrity of the system. Ideally, the formulation itself would determine which components of each system are the most important and then assign corresponding penalties for load shedding.

Reduced Generating Capacity Available (g118 offline) & Double Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	93116	97576	The following loads were shed: d107-55.7mwhr*, d110-195mwhr*.
3	FDM	4460	93116	97576	No Change, stable condition met in FDM
4	PFM	4460	93116	97576	No Change, stable condition met in PFM
Combined Model		6510	79616	86126	The following loads were shed: d103-180mwhr, d109-70.7mwhr*.

Table 6. Results from our initial iterative algorithm with reduced power generating capacity available (generator node g118 offline) and a double the original load shedding penalties on all demand nodes for each system.* Independent demand node.

3. No load shedding penalty on the interdependent demand nodes for each system.

As before, we make the load shedding penalties on interdependent demand nodes zero. Table 7 summarizes how the algorithm progresses through this scenario. Iteration 1 solves the FDM model to optimality as an isolated system. Here again, we observe that the lack of a load shedding penalty on the interdependent nodes results in a failure to fill their associated fuel demand. The FDM objective value is cut by 64.1% due to not delivering fuel to these nodes. The solution is passed as parameters to the PFM in Iteration 2. The PFM is then solved to optimality. We observe that the generators requiring fuel oil to produce power (interdependent generators) do not produce any because they did not receive fuel in the fuel model. The interdependent demand nodes (corresponding FDM components) also did not receive power because the load shedding penalties are also set to zero. We see more expensive means of power generation used to try to meet all the power demand with non-zero load-shedding penalties. Because the system does not have enough capacity to fill all the demands, it cannot avoid load shedding from the following independent demand nodes:

- *d107* shedding 125mwhr,
- *d108* shedding 171mwhr,
- *d109* shedding 104mwhr,
- *d110* shedding 195mwhr.

The reduction in demand for power, due to no power load shedding penalty for dependent nodes, pushes the objective value down, as less power needs to be generated. However, using more expensive generators and having 595 megawatt hours of power shed acts to increase the objective value. The overall effect on the PFM objective value is an increase of 13.8% to $90,579. The results are passed as parameters to the FDM. When the FDM is solved again, with all the fuel demand components in the power system receiving no power, the entire FDM is shut down and the FDM objective value reaches its max value of $135,000. In this case, three total solves are required for stable results. The total objective value and in this configuration is $225,579. Comparing the separate model solution to the combined model at reduced capacity we see the separate model is 162% from optimality of the combined value. Comparing the run with the base case we see that a non-zero load shedding penalty is required to provide the driving force for power to flow to a demand.

Reduced Generating Capacity Available (g118 offline) & No Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	1600	N/A	N/A	All interdependent fuel demand nodes loadings were shed
2	PFM	1600	90579	92179	All interdependent power demand nodes loadings were shed plus d107-125mwhr* d108-171mwhr*, d109-104mwhr*, d110-195mwhr*
3	FDM	135000	90579	225579	No fuel is delivered, all fuel demands are shed
4	PFM	135000	90579	225579	No Change, stable condition met in PFM
5	FDM	135000	90579	225579	No Change, stable condition met in FDM
Combined Model		6510	79616	86126	The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr.

Table 7. Results from our initial iterative algorithm with reduced power generating capacity available (generator node g118 offline) and no load shedding penalties on all interdependent demand nodes for each system. * Independent demand node.

F. EXCESS CAPACITY REDUCTION VIA A KNAPSACK SUBROUTINE

Our first attempt to establish dynamic interdependent demands between models aims at reducing the total objective function value by removing the use of fuel oil generators, if possible to do so, without incurring load shedding penalties. To accomplish this task, we employ a knapsack type subroutine where we use an MIP formulation to choose the most expensive subset of generators requiring fuel (*grf*) and shutting them off, while reducing the total power capacity to no less than the total demand. This subroutine forces the use of power generation from sources other than fuel oil, saving money in the transportation of fuel oil to the associated fuel demand nodes. By design, the PFM uses the cheapest available generators, thus the only possible effect on the PFM objective function is to increase or at best hold it constant.

We run the subroutine after the initial PFM solve and after each addition PFM solve. This subroutine could be run first and only once, but we would like to see how or if it changes the progression of the overall algorithm. We explore the same three cases for penalties on interdependent demand nodes with all generators available and power generation capacity greater than demand.

1. Constant and equal load shedding penalty on demand nodes per system.

Table 8 summarizes the progression of our iterative algorithm with our subroutine when used for the base case. Here we assign high and equal load shedding penalties for every demand node in each system. In iteration 1, we solve the FDM model to optimality as an isolated system. We observe that all the fuel demands were met, and the results are then passed as parameters to the PFM. In iteration 2, the PFM is solved to optimality with the results passed as parameters back to the FDM. Iteration 3 implements the subroutine which determines that shutting off the following generators will maximize the cost savings: g107a, g115a, g115b, g115c, g115d, and g115e. The subroutine shuts off or reduces the fuel demands of the associated generator that were turned off as follows: FN1 to 0 barrels/hr or 0% of the original flow and FN4 to 300 barrels/hr or 66.6% of the original flow. It also adjusts the threshold for fuel flow such that the required fuel is reduced equivalently. Iteration 4 solves the FDM again to optimality. The FDM objective value is now $3825, a reduction of 16.6%. The results are passed as parameters to the PFM. Iteration five solves the PFM to optimality with a PFM objective value of $79,710, an increase of 0.1%. The total objective value for solving the separate operator models with interdependencies using our algorithm and subroutine is now $83,535. Comparing the separate model solution to the combined model we see the separate model shrinks to 16.5% from 17.2% without the subroutine. This is an improvement of 0.7%

Full Generating Capacity Available & Constant Load Shedding Penalty on all demand nodes per system, implementing the subroutine					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	79593	84053	Optimal PFM solution in isolation, no load shedding
3	SubR				Fuel dependent generator shut off: g107a, g115a, g115b, g115c,g115d, g115e Fuel Demands changes as a % or original value: FN1-0 barrels/hr (0%), FN4-300 barrels/hr (66.6%)
4	FDM	3825	79593	83418	Optimal FDM solution in isolation (16.6% reduction in FDM objective value). No load shedding,
5	PFM	3825	79710	83535	Small increase in PFM objective value due to using more expensive generators.
6	SubR				No Change from previous subroutine run.
7	FDM	3825	79710	83535	No Change, stable condition met in FDM
8	PFM	3825	79710	83535	No Change, stable condition met in PFM
Combined Model		4735	66956	71691	Utilizing the subroutine the Separate model solution improves to **16.5% from 17.2%** from optimality. The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr. * Independent demand node.

Table 8. Results from our iterative algorithm with subroutine, all generators available, and a constant and equal load shedding penalties on all demand nodes for each system.

1. Double the load shedding penalties for interdependent nodes

Table 9 summarizes the progression of our algorithm again for the case where we increase the load shedding penalty on interdependent nodes by a factor of two, now with our subroutine for removing excess capacity by removing interdependent generators. As expected, this run progresses exactly as the previous run because the subroutine only removes excess power capacity, thus no load shedding occurs, and the results are exactly the same. Table 9 describes how the algorithm progresses.

Full Generating Capacity Available & Double Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	4460	N/A	N/A	Optimal FDM solution in isolation, no load shedding
2	PFM	4460	79593	84053	Optimal PFM solution in isolation, no load shedding
3	SubR				Fuel dependent generator shut off: g107a, g115a, g115b, g115c,g115d, g115e Fuel Demands changes as a % or original value: FN1-0 barrels/hr (0%), FN4-300 barrels/hr (66.6%)
4	FDM	3825	79593	83418	Optimal FDM solution in isolation (16.6% reduction in FDM objective value). No load shedding,
5	PFM	3825	79710	83535	Small increase in PFM objective value due to using more expensive generators.
6	SubR				No Change from previous subroutine run.
7	FDM	3825	79710	83535	No Change, stable condition met in FDM
8	PFM	3825	79710	83535	No Change, stable condition met in PFM
Combined Model		4735	66956	71691	Utilizing the subroutine the Separate model solution improves to **16.5% from 17.2%** from optimality. The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr. * Independent demand node.

Table 9. Results from our iterative algorithm with subroutine, all generators available, and double the load shedding penalties on interdependent demand nodes.

2. Zero load shedding penalties for interdependent nodes

Table 10 summarizes the progression of our algorithm detailing the case where we have zero load shedding penalties on interdependent nodes, and where we implement our subroutine for removing excess capacity by removing interdependent generators. As expected this run appears exactly as our baseline without the subroutine. The subroutine only removes excess power capacity of interdependent generators. Without a load shedding penalties, all interdependent generators will not receive fuel in the FDM and thus are inoperable in the PFM.

Full Generating Capacity Available & No Load Shedding Penalty on all Interdependent Demand Nodes					
Iteration	**Model Solved**	**Current FDM Objective Value ($)**	**Current PFM Objective Value ($)**	**Current Total Objective Value ($)**	**Notable Observations**
1	FDM	1600	N/A	N/A	All interdependent fuel demand nodes loadings were shed
2	PFM	1600	70771	72371	All interdependent power demand nodes loadings were shed plus d119-181 mwhr* & d120 -14 mwhr*
3	SubR				Fuel dependent generator shut off: g107a, g115a, g115b, g115c, g115d, g115e. Fuel Demands changes as a % or original value: FN1-0 barrels/hr (0%), FN4-300 barrels/hr (66.6%)
4	FDM	135000	70771	205771	No fuel is delivered, all fuel demands are shed
5	PFM	135000	70771	205771	No Change, stable condition met in PFM
6	FDM	135000	70771	205771	No Change, stable condition met in FDM
Combined Model		4735	66956	71691	The following loads were shed: FN4-450 barrels/hr, FN15-285 barrels/hr, FN16-535 barrels/hr, d103-180mwhr, d114-194mwhr, d115-54mwhr*, d116-100mwhr.

Table 10. Results from our iterative algorithm with subroutine, all generators available, and no load shedding penalties on interdependent demand nodes. * Independent demand node.

G. RESULTS SUMMARY

Figures 11 and 12 provide a summary of the results from both separately solved interdependent models alongside the results from solving the corresponding combined model.

Iterative Algorithm								
	All Generators Available (100% Capacity)				Generator g118 outage (86.7% Capacity)			
Penalty	**equal**	**double**	**zero**	**Monolithic**	**equal**	**double**	**zero**	**Monolithic**
number of iterations	2	2	3	1	2	2	3	1
Fuel Objective Value	4460	4460	135000	4735	4460	4460	135000	6510
Power Objective Value	79593	79593	70771	66956	93116	93116	90579	79616
Total Objective Value	84053	84053	205771	71691	97576	97576	225579	86126

Figure 11. Summary results from our Iterative Algorithm

<table>
<tr><th colspan="8">Iterative Algorithm with Subroutine</th></tr>
<tr><th></th><th colspan="4">All Generators Available (100% Capacity)</th><th colspan="3">Generator g118 outage (86.7% Capacity)</th></tr>
<tr><th>Penalty</th><th>equal</th><th>double</th><th>zero</th><th>Monolithic</th><th>equal</th><th>double</th><th>zero</th></tr>
<tr><td>number of solves</td><td>5</td><td>5</td><td>4</td><td>1</td><td colspan="3" rowspan="4">Subroutine requires excess generating capacity to affect the objective values</td></tr>
<tr><td>Fuel Objective Value</td><td>3825</td><td>3825</td><td>135000</td><td>4735</td></tr>
<tr><td>Power Objective Value</td><td>79710</td><td>79710</td><td>70771</td><td>66956</td></tr>
<tr><td>Total Objective Value</td><td>83535</td><td>83535</td><td>205771</td><td>71691</td></tr>
</table>

Figure 12. Summary results from our Iterative Algorithm with subroutine

Looking at Figures 11 and 12 we see the effectiveness in solving the full combined model. When examining solutions of the iterative algorithm on the separate operator models, it is clear that setting the load shedding penalty appropriately is crucial for determining how close the solution is to that of the combined model.

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V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Although building and solving operational models of isolated infrastructure systems allows for easier modeling and the direct application of standard, attacker-defender algorithms to optimally enhance resiliency, these types of studies ignore interdependencies between infrastructure systems, and can hide vulnerabilities to a system that involve damage to a completely different system. These studies are common and can reveal important information about weaknesses and vital components which should be hardened, but our work seeks to expand the space of study to explicitly account for interdependencies in the operational models.

We modeled two well-known systems and validated their behavior and responses to different scenarios using two different formulations. From there, we modeled their interdependencies and generated a scenario for our case study to allow us to demonstrate the feasibility of our technique on a medium-sized and medium-complexity pair of systems. We compared these solutions against a monolithic “combined” model, including all the components for both systems, which established a true baseline for optimal performance. We also explored a family of iterative algorithms based on setting policies for load shedding penalties to interdependent infrastructure demand nodes, and found feasible solutions of varying quality, some of which were within 16% of the true optimum.

B. RECOMMENDATIONS

The next phase in this research should focus on mechanisms to communicate demands, and, more specifically, the consequences of not satisfying those demands, more directly between the separate models of the individual infrastructures. We began to address this with the simple subroutine that evaluated extra capacity and directly shutoff any “requirements” for excess fuel. Although this specific subroutine only works with excess capacity, it does work to change demand and associated load shedding penalties.

The FDM can be improved by adding more details and specific components that were implied in our work but never actually formally inserted (i.e., pumps on arcs that require electrical power to allow flow). This may allow for a subroutine that could compare costs and benefits of turning on a pump without having to reference the PFM directly, possibly using an estimate of average power cost for that pump or a heuristic estimate based on more detailed engineering models.

We effectively built, tested, and began to explore methods to have the separate systems pass information back and forth and solve to feasible solutions. More work is required in this area to get acceptably close to the optimal solution. The first step should be to investigate the use of dynamic penalties for load shedding to dependent nodes; these could be updated at each iteration based on requirements in the corresponding system. This might be accomplished by in the PFM by taking an average of the cost of power generated during each iterative solve. The second step would be to establish more realistic power demands for fuel components by increasing the detail of the FDM (pumps, heater, agitators, etc.). This will give more flexibility in the types of dependences that could be demonstrated but also help ensure a truer balance within the total objective function.

APPENDIX A. GAMS CODE

A. SHARED ELEMENTS OF THE PFM AND FDM

Sets

```
fp complete set of all Fuel and Power nodes/
$include power.nodes.txt
$include fuel.nodes.txt
/
;
alias(fp,fpi,fpj)
;
set
fn(fp) fuel nodes /
$include fuel.nodes.txt
/;
alias(fn,fi,fii,fj,fjj)
;
Sets
pn(fp) power nodes /
$include power.nodes.txt
/;
alias(pn,pi,pj)
;
sets
d(pn) power demand nodes /
$include power.demands.txt
/
i(pn) power buse nodes /
$include power.buses.txt
/
PArcs(pn,pn) power arcs
/
$ondelim
$include power.arcs.csv
$offdelim
/
FArcs(fn,fn) fuel arcs
/
$ondelim
$include fuel.arcs.csv
$offdelim
/
pdn(d) power depend nodes
/
```

```
$include power.depend.node.txt
/
fdn(fn) fuel depend nodes
/
$include fuel.depend.node.txt
/
g(pn) power generator nodes /
$include power.generating_units.txt
/
fd(fn) fuel demand nodes
/
$include fuel.demand.nodes.txt
/
;
parameter PDem(d) Demand of Power Node d/
$ondelim
$include power.load_consumer.data.csv
$offdelim
/;
table PArcCap(pi,pj) power acr capacity of pi_pj
$ondelim
$include power.line.capacity.data.csv
$offdelim
;
Set grf(g)   generators that require fuel oil from FDM
/
$include gen.req.fuel.node.txt
/
dfd(fd)  fuel nodes that are generators in the power
network
/
$include dependent.fuel.node.txt
/
gnfd(grf,dfd) generator node and coresponding fuel
node
/
$ondelim
$include gen.and.fuel.nodes.csv
$offdelim
/
;
Positive Variable
FY(fi,fj)    Flow in arc fi fj in barrels
PY_pos(pn,pn) power flow on line l (MW)
PY_neg(pn,pn) power flow on line l (MW)
;
```

```
Binary variable
Turn_Off_FDem(grf)
;
loop(grf,
Turn_Off_FDem.l(grf)=0;)
;
Scalar zp /0/
;
```

B. FUEL ELEMENTS

```
set
Fedge(fn,fn) fuel edges
*Establish Fuel Edges via NPS convention
PDFN(i,d,fi,fj)     fuel arcs that depend on power
/
$ondelim
$include PowerIntDep.csv
$offdelim
/;
loop(farcs(fi,fj)$(ord(fi) < ord(fj)),
  Fedge(fi,fj)=yes;)
;
set
fs(fn) fuel supply nodes
/
$include fuel.supply.nodes.txt
/
;
table fuel_arc_data(fi,fj,*) FArcCost(cost)
FArcCap(cap)  FArcCostDam(cost_dam)
$ondelim
$include fuel.arc.data.csv
$offdelim
;
Table fuel_node_data(fn,*) FSupply(supply)
FLdSCost(penalty)
$ondelim
$include fuel.node.data.csv
$offdelim
;
Parameter PThreshold(pdn) Power required for node fpn
in the fuel network to function;
loop(pdn,
PThreshold(pdn)= PDem(pdn);)
```

```
;
positive variable
FLdS(fp)         fuel shortfall at node n
;
binary variable
FT(i,pdn,fi,fn)   Binary switch turning capcity of
fi_fn on 1 or off 0
;
Variable
fz
;
Equations
  FObj
  FuelNodeBalance(fn)
  FuelMinFlow(fi,fj)
  FuelMaxFlow(fi,fj)
  PowerDepFuel_FM(i,pdn,fi,fj)
  FuelDepGen_FM(i,pdn,fi,fn)
  FLdS_UB(fn)
  NoFLdSForSupplier(fn)
;
*Fuel network objective
  FObj..
    fz =e= sum((FArcs(fi,fj)),
fuel_arc_data(fi,fj,'cost')*FY(fi,fj)+

fuel_arc_data(fi,fj,'cost_dam')*fuel_arc_data(fi,fj,'x
')*FY(fi,fj))
         +
sum(fn,fuel_node_data(fn,'penalty')*FLdS(fn))
  ;
  FuelNodeBalance(fn)..
    sum(FArcs(fn,fj), FY(fn,fj))-
sum(FArcs(fj,fn),FY(fj,fn))-FLdS(fn)=l=
          fuel_node_data(fn,'supply')
  ;
  FuelMinFlow(fi,fj)$FArcs(fi,fj)..
    0 =L= FY(fi,fj) + FY(fj,fi)
  ;
  FuelMaxFlow(fi,fj)$FArcs(fi,fj)..
    FY(fi,fj) + FY(fj,fi) =L=
fuel_arc_data(fi,fj,'cap')
  ;
  PowerDepFuel_FM(i,pdn,fi,fj)$PDFN(i,pdn,fi,fj)..
    FY(fi,fj)  =L=
fuel_arc_data(fi,fj,'cap')*FT(i,pdn,fi,fj)
```

```
*- FY(fj,fi)
  ;
  FuelDepGen_FM(i,pdn,fi,fj)$PDFN(i,pdn,fi,fj)..
     PThreshold(pdn)*FT(i,pdn,fi,fj) =l=
Sum(PArcs(pn,pdn),PY_pos(pn,pdn)-PY_neg(pn,pdn))
  ;
  FLdS_UB(fd)..
     FLdS(fd) =l=  -fuel_node_data(fd,'supply')
  ;
  NoFLdSForSupplier(fs)..
     FLdS(fs)=e=0
  ;
parameter FRec(fd)
;
MODEL FuelOperatorsModel /
FObj
FuelNodeBalance
FuelMinFlow
FuelMaxFlow
PowerDepFuel_FM
FuelDepGen_FM
FLdS_UB
NoFLdSForSupplier
/
;
```

C. FUEL INITIAL EXECUTE

```
loop(PArcs(pi,pj),
PY_pos.l(pi,pj)=PArcCap(pi,pj);
PY_neg.l(pi,pj)=0;)
;
```

D. FUEL EXECUTE

```
loop(PArcs(pi,pj),
PY_pos.fx(pi,pj)=PY_pos.l(pi,pj);
PY_neg.fx(pi,pj)=PY_neg.l(pi,pj);
);
loop(fdn,
fuel_node_data(fdn,'penalty')=0 ;)
;
SOLVE FuelOperatorsModel USING MIP MINIMIZING fz
;
loop(PArcs(pi,pj),
PY_pos.lo(pi,pj)=0;
PY_pos.up(pi,pj)=PArcCap(pi,pj);
```

```
PY_neg.lo(pi,pj)=0;
PY_neg.up(pi,pj)=PArcCap(pi,pj);)
;
loop(fdn,
fuel_node_data(fdn,'penalty')=150 ;)
;
Display fz.l, FY.l, FLds.l,FT.l;
loop(fd,
FRec(fd) =  -(FLdS.l(fd) +
fuel_node_data(fd,'supply'));)
;
Display FRec
;
```

E. POWER ELEMENTS

```
set
FDPN(g,i,fi,fn)     generator arcs that depend on fuel
/
$ondelim
$include FuelIntDep.csv
$offdelim
/;
parameter PGenCap(g) power_generator_capacity_data  /
$ondelim
$include power.generator.capacity.data.csv
$offdelim
/;
parameter  PGenCost(g)  power generator cost per MW by
node g/
$ondelim
$include power.generation.cost.data.csv
$offdelim
/;
parameter PLdSCost(d) power load shedding cost of node
d/
$ondelim
$include power.load.shedding.cost.data.csv
$offdelim
/;
table PArcRes(pi,pj) resistance of arc pi_pj
$ondelim
$include power.resistance.line.data.csv
$offdelim
;
table PArcRea(pi,pj) reactance of arc pi_pj
```

```
$ondelim
$include power.reactance.line.data.csv
$offdelim
;
parameter   Bi(pn,pn) susceptance
;
Loop(PArcs(pi,pj),
  Bi(pi,pj) = PArcRea(pi,pj) /
(PArcRea(pi,pj)*PArcRea(pi,pj)+PArcRes(pi,pj)*PArcRes(
pi,pj)))
;
Set
PEdge(pn,pn) electric edges
;
parameter
FThreshold(fdn)    Fuel required for node pdn in the
power network to function
;
loop(fdn,
FThreshold(fdn) = -fuel_node_data(fdn,'supply');)
;
loop(PArcs(pi,pj)$(ord(pi) < ord(pj)),
  PEdge(pi,pj)=yes;)
;
Variable
  pz objective value
  theta(pn) phase angle at bus i
;
positive variables
  PGen(g) power generated by g (MW)
  PLdS(d) load shed by customer sector c at bus i (MW)
;
Binary Variable
PT(g,i,fi,fdn)
;
Equations
   PObj
   PowerFlowOnLine(pn,pn)
   PowerBusBalance(i)
   PowerGenBalance(g)
   PowerDemBalance(d)
   PowerOneArc(pi,pj)
   PowerMinPowerOnLine(pn,pn)
   PowerMinPowerOnLineT(g,i,fi,fdn)
   PowerMaxPowerOnLine(pn,pn)
   PowerMaxPowerOnLineT(g,i,fi,fdn)
```

```
   PowerMaxPowerProduction(g)
   PowerMaxPowerProductDep(grf)
   PowerLoadShed(d)
   Threshold_PM(g,i,fi,fdn)
   TotalDemand
;
 PObj..
   pz =g=
sum(g,PGenCost(g)*PGen(g))+sum(d,PLdSCost(d)*PLdS(d))
;
 PowerFlowOnLine(pi,pj)$(PEdge(pi,pj))..
* XX(b,bb)* Pline(b,bb) =e=
Bi(b,bb)*(XX(b,bb)*(theta(bb) - theta(b)))
  (PY_pos(pi,pj)-PY_neg(pi,pj)) =e=
Bi(pi,pj)*(theta(pj) - theta(pi))
;
 PowerBusBalance(i)..
   sum(PArcs(pn,i),(PY_pos(pn,i)-PY_neg(pn,i)))=e= 0
;
 PowerGenBalance(g)..
   PGen(g) - sum(PArcs(g,pn), (PY_pos(g,pn)-
PY_neg(g,pn)) ) =e= 0
;
 PowerDemBalance(d)..
   sum(PArcs(pn,d),(PY_pos(pn,d)-PY_neg(pn,d))) +
PLdS(d)=e= PDem(d)
;
 PowerOneArc(pi,pj)$PArcs(pi,pj)..
   (PY_pos(pi,pj)-PY_neg(pi,pj)) =e= -(PY_pos(pj,pi)-
PY_neg(pj,pi))
;
 PowerMinPowerOnLine(pi,pj)$PArcs(pi,pj)..
  -1 * PArcCap(pi,pj) =l= (PY_pos(pi,pj)-
PY_neg(pi,pj))
*  -1 * XX(b,bb)*PbarLine(b,bb) =l= Pline(b,bb)
;
 PowerMinPowerOnLineT(g,i,fi,fdn)$FDPN(g,i,fi,fdn)..
  -1 * PArcCap(g,i)*PT(g,i,fi,fdn) =l= (PY_pos(g,i)-
PY_neg(g,i))
*  -1 * XX(b,bb)*PbarLine(b,bb) =l= Pline(b,bb)
;
 PowerMaxPowerOnLine(pi,pj)$PArcs(pi,pj)..
   (PY_pos(pi,pj)-PY_neg(pi,pj)) =l= PArcCap(pi,pj)*1
*   Pline(b,bb) =l= PbarLine(b,bb)*XX(b,bb)
;
 PowerMaxPowerOnLineT(g,i,fi,fdn)$FDPN(g,i,fi,fdn)..
```

```
   (PY_pos(g,i)-PY_neg(g,i)) =l=
PArcCap(g,i)*PT(g,i,fi,fdn)
*    Pline(b,bb) =l= PbarLine(b,bb)*XX(b,bb)
;

 PowerMaxPowerProductDep(grf)..
   PGen(grf)=l= PGenCap(grf)*(1-Turn_Off_FDem.L(grf))
;



 PowerMaxPowerProduction(g)..
   PGen(g) =l= PGenCap(g)
;
 PowerLoadShed(d)..
   PLdS(d) =l= PDem(d)
;
 Threshold_PM(g,i,fi,fdn)$FDPN(g,i,fi,fdn)..
   FThreshold(fdn)*PT(g,i,fi,fdn) =l=
sum(FArcs(fn,fdn),FY(fn,fdn))-
Sum(FArcs(fdn,fn),FY(fdn,fn))
;
 TotalDemand..
   Sum(d,PDem(d))=e=Sum(g,PGen(g))+sum(d,PLdS(d))
;
MODEL PowerOperatorsModel /
PObj
PowerFlowOnLine
PowerBusBalance
PowerGenBalance
PowerDemBalance
PowerOneArc
PowerMinPowerOnLine
PowerMinPowerOnLineT
PowerMaxPowerOnLine
PowerMaxPowerOnLineT
PowerMaxPowerProduction
PowerMaxPowerProductDep
PowerLoadShed
Threshold_PM
TotalDemand
/
;
```

F. POWER EXECUTE

```
Loop(FArcs(fi,fj),
FY.fx(fi,fj)=FY.l(fi,fj);)
;
loop(pdn,
PLdSCost(pdn)=0;)
;
PGenCap('g118')=0
;
SOLVE PowerOperatorsModel USING MIP MINIMIZING pz
;
loop(pdn,
PLdSCost(pdn)=75;)
;
Loop(FArcs(fi,fj),
FY.lo(fi,fj)=0;
FY.up(fi,fj)=fuel_arc_data(fi,fj,'cap');)
;
scalars PTotalGenCap, PTotalDem
;
PTotalGenCap=sum(g,PGen.l(g))
;
PTotalDem=sum(d,PDem(d))
;
Display   pz.l,PY_pos.l,PY_neg.l, theta.l,PGen.l,
PLdS.l,PT.l,FY.l,FThreshold,PTotalGenCap,PTotalDem
;
```

G. SUBROUTINE ELEMENTS

```
Scalar MWhr_to_BarrelsPerHr /1.875/
;
variable
  fsz  objective
;
equation
  FSubObj
  CapGenOff
;
FSubObj..
    fsz =l=
sum(grf,PGenCost(grf)*PGenCap(grf)*Turn_Off_FDem(grf))
;
CapGenOff..
```

```
    sum(grf, PGenCap(grf)*Turn_Off_FDem(grf)) =l=
sum(g,PGenCap(g))-Sum(d,PDem(d))
;
Display grf
;
Model FuelDemSubRout /
FSubObj
CapGenOff
/
;
```

H. SUBROUTINE EXECUTION

```
Solve FuelDemSubRout using MIP maximizing fsz
;
Display CapGenOff.l,Turn_Off_FDem.l
;
loop(dfd,
fuel_node_data(dfd,’supply’)=-Sum(gnfd(grf,dfd),(1-
Turn_Off_FDem.l(grf))*PGenCap(grf)*MWhr_to_BarrelsPerH
r);)
;
loop(fdn$(-fuel_node_data(fdn,’supply’) gt 0),
FThreshold(fdn) = -fuel_node_data(fdn,’supply’);)
;
Display  fuel_node_data, FThreshold
;
```

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APPENDIX B. DATA TABLES

1. fuel.nodes.txt

FN1	FN5	FN10	FN14
FN2	FN6	FN10p	FN15
FN2p	FN7	FN11	FN16
FN3	FN8	FN12	
FN4	FN8p	FN12p	
FN4p	FN9	FN13	

2. power.nodes.txt

i101	i117	g102c	g121	d108
i102	i118	g102d	g122a	d109
i103	i118p	g107a	g122b	d110
i104	i119	g107b	g122c	d113
i105	i119p	g107c	g122d	d114
i106	i120	g113a	g122e	d115
i107	i120p	g113b	g122f	d116
i108	i121	g113c	g123a	d118
i109	i122	g114	g123b	d119
i110	i123	g115a	g123c	d120
i111	i124	g115b	d101	
i112	g101a	g115c	d102	
i113	g101b	g115d	d103	
i114	g101c	g115e	d104	
i115	g101d	g115f	d105	
i115p	g102a	g116	d106	
i116	g102b	g118	d107	

3. power.demands.txt

d101	d106	d113	d119
d102	d107	d114	d120
d103	d108	d115	
d104	d109	d116	
d105	d110	d118	

4. power.buses.txt

i101	i103	i105	i107
i102	i104	i106	i108

i109
i110
i111
i112
i113
i114
i115
i115p
i116
i117
i118
i118p
i119
i119p
i120
i120p
i121
i122
i123
i124

5. power.arcs.csv

d101	i101
d102	i102
d103	i103
d104	i104
d105	i105
d106	i106
d107	i107
d108	i108
d109	i109
d110	i110
d113	i113
d114	i114
d115	i115
d116	i116
d118	i118
d119	i119
d120	i120
g101a	i101
g101b	i101
g101c	i101
g101d	i101
g102a	i102
g102b	i102
g102c	i102
g102d	i102
g107a	i107
g107b	i107
g107c	i107
g113a	i113
g113b	i113
g113c	i113
g114	i114
g115a	i115
g115b	i115
g115c	i115
g115d	i115
g115e	i115
g115f	i115
g116	i116
g118	i118
g121	i121
g122a	i122
g122b	i122
g122c	i122
g122d	i122
g122e	i122
g122f	i122
g123a	i123
g123b	i123
g123c	i123
i101	d101
i101	g101a
i101	g101b
i101	g101c
i101	g101d
i101	i102
i101	i103
i101	i105
i102	d102
i102	g102a
i102	g102b
i102	g102c
i102	g102d
i102	i101
i102	i104
i102	i106
i103	d103
i103	i101
i103	i109
i103	i124
i104	d104
i104	i102
i104	i109
i105	d105
i105	i101
i105	i110
i106	d106
i106	i102
i106	i110
i107	d107
i107	g107a
i107	g107b
i107	g107c
i107	i108
i108	d108
i108	i107
i108	i109
i108	i110
i109	d109
i109	i103
i109	i104
i109	i108
i109	i111
i109	i112
i110	d110
i110	i105
i110	i106
i110	i108
i110	i111

i110	i112
i111	i109
i111	i110
i111	i113
i111	i114
i112	i109
i112	i110
i112	i113
i113	d113
i113	g113a
i113	g113b
i113	g113c
i113	i111
i113	i112
i113	i123
i114	d114
i114	g114
i114	i111
i114	i116
i115	d115
i115	g115a
i115	g115b
i115	g115c
i115	g115d
i115	g115e
i115	g115f
i115	i115p
i115	i116

i115	i121
i115	i124
i115p	i115
i115p	i121
i116	d116
i116	g116
i116	i114
i116	i115
i116	i117
i116	i119
i117	i116
i117	i118
i117	i122
i118	d118
i118	g118
i118	i117
i118	i118p
i118	i121
i118p	i118
i118p	i121
i119	i116
i119	d119
i119	i119p
i119	i120
i119p	i119
i119p	i120
i120	i119
i120	d120

i120	i119p
i120	i120p
i120	i123
i120p	i120
i120p	i123
i121	g121
i121	i115
i121	i115p
i121	i118
i121	i118p
i121	i122
i122	g122a
i122	g122b
i122	g122c
i122	g122d
i122	g122e
i122	g122f
i122	i117
i122	i121
i123	g123a
i123	g123b
i123	g123c
i123	i113
i123	i120
i123	i120p
i124	i103
i124	i115

6. fuel.arcs.csv

FN1	FN2
FN1	FN5
FN2	FN1
FN2	FN3
FN2	FN2p
FN2p	FN3
FN2	FN7
FN3	FN2p
FN2p	FN2
FN3	FN2

FN4	FN8
FN4	FN4p
FN4p	FN8
FN5	FN1
FN5	FN9
FN6	FN7
FN6	FN10
FN7	FN2
FN7	FN6
FN7	FN8

FN8	FN4
FN8	FN4p
FN8p	FN8
FN4p	FN4
FN8	FN7
FN8	FN12
FN9	FN5
FN9	FN13
FN10	FN6
FN10	FN11

FN10	FN13
FN10p	FN10
FN11	FN10
FN11	FN12
FN11	FN15
FN12	FN8
FN12	FN11

FN12	FN12p
FN12p	FN16
FN12	FN16
FN13	FN9
FN13	FN10
FN13	FN14
FN14	FN13

FN14	FN15
FN15	FN11
FN15	FN14
FN16	FN12
FN16	FN12p
FN12p	FN12

7. Power.depend.node.txt

d103
d105
d114
d116

8. fuel.demand.node.txt

FN1
FN2
FN2p
FN3
FN4
FN4p
FN5
FN6
FN7
FN9
FN11
FN12
FN12p
FN13
FN14
FN15
FN16

9. Power.generating_units.txt

g101a
g101b
g101c
g101d
g102a
g102b
g102c
g102d
g107a
g107b
g107c
g113a
g113b
g113c
g114
g115a
g115b
g115c
g115d
g115e
g115f
g116
g118
g121
g122a
g122b
g122c
g122d
g122e
g122f
g123a
g123b
g123c

10. fuel.depend.nodes.txt

fn1
fn4
fn13
fn15
fn16

11. power.load_consumer.data.csv

d101	108
d102	97
d103	180
d104	74
d105	71
d106	136

d107	125
d108	171
d109	175
d110	195
d113	265
d114	194

d115	317
d116	100
d118	333
d119	181
d120	128

12. power.line.capacity.data.csv

All arc capacities is set to 1000.

13. gen.req.fuel.node.txt

g101c
g101d
g102c
g102d
g107a

g107b
g107c
g113a
g113b
g113c

g115a
g115b
g115c
g115d
g115e

14. dependent.fuel.node.txt

fn1
fn4
fn13
fn15
fn16

15. gen.and.guel.nodes.csv

g101c	fn15
g101d	fn15
g102c	fn13
g102d	fn13
g107a	fn4

g107b	fn4
g107c	fn4
g113a	fn16
g113b	fn16
g113c	fn16

g115a	fn1
g115b	fn1
g115c	fn1
g115d	fn1
g115e	fn1

16. PowerIntDep.csv

i103	d103	fn10	fn6
i103	d103	fn10	fn11
i105	d105	fn10	fn13
i105	d105	fn10p	fn10
i114	d114	fn8p	fn8
i114	d114	fn8	fn4p
i116	d116	fn8	fn4
i116	d116	fn8	fn7
i116	d116	fn8	fn12

17. fuel.supply.nodes.txt

FN8
FN8p
FN10
FN10p

18. fuel.arc.data.csv

tail	head	cap	cost	cost_dam	x	t
FN1	FN2	1350	1	10	0	1
FN1	FN5	1350	1	10	0	1
FN2	FN1	1350	1	10	0	1
FN2	FN3	1350	1	10	0	1
FN2	FN2p	1350	0	0	0	1
FN2p	FN3	1350	1	10	0	1
FN2	FN7	1350	1	10	0	1
FN3	FN2p	1350	0	0	0	1
FN2p	FN2	1350	1	10	0	1
FN3	FN2	1350	1	10	0	1
FN4	FN8	1350	1	10	0	1
FN4	FN4p	1350	0	0	0	1
FN4p	FN8	1350	1	10	0	1
FN5	FN1	1350	1	10	0	1
FN5	FN9	1350	1	10	0	1
FN6	FN7	1350	1	10	0	1
FN6	FN10	1350	1	10	0	1
FN7	FN2	1350	1	10	0	1
FN7	FN6	1350	1	10	0	1
FN7	FN8	1350	1	10	0	1
FN8	FN4	1350	1	10	0	1

FN8	FN4p	1350	0	0	0	1
FN8p	FN8	1350	0.0001	0	0	1
FN4p	FN4	1350	1	10	0	1
FN8	FN7	1350	1	10	0	1
FN8	FN12	1350	1	10	0	1
FN10p	FN10	1350	0.0001	0	0	1
FN9	FN5	1350	1	10	0	1
FN9	FN13	1350	1	10	0	1
FN10	FN6	1350	1	10	0	1
FN10	FN11	1350	1	10	0	1
FN10	FN13	1350	1	10	0	1
FN11	FN10	1350	1	10	0	1
FN11	FN12	1350	1	10	0	1
FN11	FN15	1350	1	10	0	1
FN12	FN8	1350	1	10	0	1
FN12	FN11	1350	1	10	0	1
FN12	FN12p	1350	0	0	0	1
FN12p	FN16	1350	1	10	0	1
FN12	FN16	1350	1	10	0	1
FN13	FN9	1350	1	10	0	1
FN13	FN10	1350	1	10	0	1
FN13	FN14	1350	1	10	0	1
FN14	FN13	1350	1	10	0	1
FN14	FN15	1350	1	10	0	1
FN15	FN11	1350	1	10	0	1
FN15	FN14	1350	1	10	0	1
FN16	FN12	1350	1	10	0	1
FN16	FN12p	1350	0	0	0	1
FN12p	FN12	1350	1	10	0	1

19. Fuel.node.data.csv

node	supply	penalty
FN1	-112.5	150
FN2	-100	150
FN2p	0	0
FN3	-100	150
FN4	-450	150
FN4p	0	0
FN5	-100	150
FN6	-100	150
FN7	-100	150
FN8	0	150
FN8P	1350	0
FN9	-100	150
FN10	0	150
FN10P	1350	0
FN11	-100	150
FN12	-100	150
FN12p	0	0

FN13	-285	150
FN14	-100	150

FN15	-285	150
FN16	-535	150

20. FuelIntDep.csv

g101c	i101	fn11	fn15
g101d	i101	fn11	fn15
g102c	i102	fn9	fn13
g102d	i102	fn9	fn13
g107a	i107	fn8	fn4
g107b	i107	fn8	fn4
g107c	i107	fn8	fn4
g101c	i101	fn14	fn15
g101d	i101	fn14	fn15
g102c	i102	fn14	fn13
g102d	i102	fn14	fn13
g107a	i107	fn8	fn4p
g107b	i107	fn8	fn4p
g107c	i107	fn8	fn4p
g113a	i113	fn12	fn16
g113b	i113	fn12	fn16

g113c	i113	fn12	fn16
g115a	i115	fn2	fn1
g115b	i115	fn2	fn1
g115c	i115	fn2	fn1
g115d	i115	fn2	fn1
g115e	i115	fn2	fn1
g113a	i113	fn12p	fn16
g113b	i113	fn12p	fn16
g113c	i113	fn12p	fn16
g115a	i115	fn5	fn1
g115b	i115	fn5	fn1
g115c	i115	fn5	fn1
g115d	i115	fn5	fn1
g115e	i115	fn5	fn1
g102c	i102	fn10	fn13
g102d	i102	fn10	fn13

21. Power.generator.capacity.data.csv

g101a	10
g101b	10
g101c	76
g101d	76
g102a	10
g102b	10
g102c	76
g102d	76
g107a	80
g107b	80
g107c	80

g113a	95.1
g113b	95.1
g113c	95.1
g114	0
g115a	12
g115b	12
g115c	12
g115d	12
g115e	12
g115f	155
g116	155

g118	400
g121	400
g122a	50
g122b	50
g122c	50
g122d	50
g122e	50
g122f	50
g123a	155
g123b	155
g123c	350

22. Power.generation.cost.data.csv

g101a	35.67
g101b	35.67

g101c	31.89
g101d	31.89

g102a	35.67
g102b	35.67

g102c	31.89
g102d	31.89
g107a	31.89
g107b	31.89
g107c	31.89
g113a	31.89
g113b	31.89
g113c	31.89
g114	0

g115a	31.89
g115b	31.89
g115c	31.89
g115d	31.89
g115e	31.89
g115f	31.89
g116	31.89
g118	25.48
g121	25.48

g122a	11.34
g122b	11.34
g122c	11.34
g122d	11.34
g122e	11.34
g122f	11.34
g123a	31.89
g123b	31.89
g123c	31.89

23. power.load.shedding.cost.data.csv

d101	75
d102	75
d103	75
d104	75
d105	75
d106	75

d107	75
d108	75
d109	75
d110	75
d113	75
d114	75

d115	75
d116	75
d118	75
d119	75
d120	75

24. power.resistance.line.data.csv

Resistance on all arcs is set to 5

25. power.reactance.line.data.csv

Reactance on all arcs is set to 1

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